

CE2253-APPLIED HYDRAULIC ENGINEERING

(FOR IV – SEMESTER)

UNIT - I

OPEN CHANNEL FLOW

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UNIT – I

OPEN CHANNEL FLOW

OPEN CHANNEL FLOW - TYPES AND REGIMES OF FLOW - VELOCITY
DISTRIBUTION IN OPEN CHANNEL – WIDE OPEN CHANNEL – SPECIFIC ENERGY –
CRITICAL FLOW AND ITS COMPUTATION.

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APPLIED HYDRAULIC ENGINEERING
UNIT – I
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OPEN CHANNEL FLOW - TYPES AND REGIMES OF FLOW - VELOCITY DISTRIBUTION IN OPEN CHANNEL – WIDE OPEN CHANNEL – SPECIFIC ENERGY – CRITICAL FLOW AND ITS COMPUTATION.

Two Marks Questions and Answers

1. Define open channel.

A liquid flowing at atmospheric pressure through a passage is known as flow in open channels. The flow of water through pipes at atmospheric pressure or when the level of water in the pipe is below the top of the pipe, is also classified as open channel flow.

2. What are the classifications of flow in an open channel?

1. Steady and unsteady flow
2. Uniform flow and non-uniform flow
3. Laminar flow and turbulent flow
4. Sub-critical, critical, and super critical flow

3. Define steady flow and unsteady flow.

Steady Flow

If the flow characteristics such as depth of flow, velocity of flow, rate of flow at any point in open channel flow do not change with respect to time, the flow is said to be steady flow.

$$\frac{\partial v}{\partial t} = 0, \quad \frac{\partial Q}{\partial t} = 0 \quad \text{or} \quad \frac{\partial y}{\partial t} = 0$$

Unsteady Flow

If at any point in open channel flow, the velocity of flow, depth of flow or rate of flow at any point in open channel flow changes with respect to time, the flow is said to be steady flow.

$$\frac{\partial v}{\partial t} \neq 0, \text{ or } \frac{\partial Q}{\partial t} \neq 0 \quad \text{or} \quad \frac{\partial y}{\partial t} \neq 0$$

4. Define Uniform flow and Non-Uniform flow.

Uniform flow

If for a given length of the channel, the velocity of flow, depth of flow, slope of the channel and cross-section remain constant, the flow is said to be uniform.

$$\frac{\partial v}{\partial S} = 0, \quad \frac{\partial y}{\partial S} = 0 \quad \text{for uniform flow}$$

Non – uniform flow

If for a given length of the channel, the velocity of flow, depth of flow, slope of the channel and cross-section do not remain constant, the flow is said to be non - uniform flow.

$$\frac{\partial v}{\partial S} \neq 0, \quad \frac{\partial y}{\partial S} \neq 0 \quad \text{for non-uniform flow}$$

5. What is rapidly varied flow?

It is defined as that flow in which depth of flow changes abruptly over a small length of the channel.

6. What is gradually varied flow?

If the depth of flow in a channel changes gradually over a long length of the channel, the flow is said to be gradually varied flow.

7. What is laminar and turbulent flow?

Laminar flow

The flow in open channel is said to be laminar if the Reynolds number (R_e) is than 500 or 600

$$\text{Reynolds number} = \frac{\rho V R}{\mu}$$

Turbulent flow

If the Reynolds number is more than 2000, the flow is said to be turbulent in open channel flow.

8. What is TRANSITION state?

If the R_e lies between 500 to 2000, the flow is considered to be in transition state.

9. Give a brief note on Sub-critical, Critical, Super critical flow.

Sub critical flow:

The flow in open channel is said to be sub-critical if the Froude number is less than 1.

$$F_e = \frac{V}{\sqrt{gD}}$$

Critical Flow:

The flow in open channel is said to be critical if the Froude number is 1.

Super critical flow:

The flow in open channel is said to be super critical if the Froude number is greater than

10. Give the formula relating to velocity and discharge in chezy's formula.

$$\text{Velocity} \quad V = C \sqrt{mi}$$

$$\text{Discharge} \quad Q = A C \sqrt{mi}$$

11. Give the BAZIN, GANGUILLET-KUTTER, MANNINGS formulas for chezy's constant.

a) Bazin formula

$$C = \frac{157.6}{1.81 + \frac{K}{\sqrt{m}}}$$

b) ganguillet-kutter formula

$$C = \frac{23 + \frac{0.00155}{i} + \frac{1}{N}}{1 + (23 + \frac{0.00155}{i}) \frac{N}{\sqrt{m}}}$$

c) Manning's formula

$$C = \frac{1}{N} m^{\frac{1}{6}}$$

12. Give the formula for total energy.

$$\text{TOTAL ENERGY} = z + h + \frac{V^2}{2g}$$

13. Define specific energy.

It is defined as energy per unit weight of the liquid with respect to the bottom of the channel.

14. How do you calculate specific energy?

$$E = h + \frac{V^2}{2g}$$

h = depth of liquid

V = Mean velocity of flow

g = Acceleration due to gravity.

15. What is specific energy curve?

It is defined as the curve which shows variation of specific energy with depth of flow

16. Define critical depth.

It is defined as the depth of flow of water at which the specific energy is minimum. The depth of flow of water at C is known as critical depth.

$$h_c = \left(\frac{q^2}{g} \right)^{\frac{1}{3}}$$

17. What is critical velocity?

The velocity of flow at critical depth is known as critical velocity.

$$V_c = \sqrt{gh_c}$$

Where,

V_c = Critical velocity

h_c = Critical depth

g = acceleration due to gravity

18. Represent minimum specific energy in terms of critical depth.

$$E_{\min} = \frac{3h_c}{2}$$

Where,

h_c = critical depth

E_{\min} = minimum specific energy

19. What is critical flow?

It is defined as that flow at which the specific energy is minimum or the flow corresponding to critical depth is defined as critical flow.

$$V_c = \sqrt{gh_c} \quad (\text{or}) \quad \frac{V_c}{\sqrt{gh_c}} = 0$$

20. Define streaming flow. What it is otherwise called as?

When the depth of flow in a channel is greater than the critical depth (h_c), the flow is said to be sub-critical flow (or) streaming flow (or) tranquil flow. [$F_e < 1.0$].

21. When does a super critical flow occur?

When the depth of flow in a channel is less than the critical depth (h_c), the flow is said to be super-critical flow (or) shooting flow (or) the torrential flow. ($F_e > 1.0$)

22. What is alternate depth?

In the specific energy curve, the point C corresponds to the minimum specific energy and the depth of flow at C is called critical depth. The depths corresponding to points G & H are called alternate depth.

23. Comparison between open channel flow and pipe flow

S.No	Aspects	Open channel flow	Pipe flow
1	Causes of flow	Gravity flow	Hydraulic pressures
2	Geometric of sections	May have any shape	Generally round in cross sections
3	Surface roughness	Varies widely with depth of flow	Depending upon the material of the

			pipe
4	Piezometric head	$Z + Y = \text{HGL}$ Coincides with the water surface	$Z + P/W = \text{HGL}$ does not coincide with water surface.
5	Velocity distribution	Maximum velocity occurs at a little distance below the water surface.	Maximum at center of pipe and at pipe wall $V = 0$
6	Law	Obeys frouds law inertia force /gravity force.	Obeys Reynolds law inertia force/viscous force.

24. What are the types of channels?

The types are:

- i) Natural surface: It has irregular sections of varying shapes. Ex. Rivers, streams etc.
- ii) A channel without any cover at top is known as open channel .Ex. Irrigation channels
- iii) Prismatic channel: A channel with constant beds slope and the same cross section along its length.
- iv) Experimental: The cross section of the channel is proportional to any power of depth of flow in channel. Ex;Rectangular,Triangular
- v) Non Exponential: Trapezoidal and circular channel are non exponential channels.

25. What are regimes of flow?

Regimes of flow are the result of joint influence of viscosity and gravity. The four common stages of flow, viz, laminar, turbulent, sub critical and super critical flows.

TYPES:

- 1) Sub critical laminar
- 2) Super critical laminar
- 3) Sub critical turbulent
- 4) Super critical turbulent.

26) What is wide open channel?

If the width of the channel is equal to or more than ten times the depth of flow it may be called as wide open channel. For experimental or analytical purposes the flow in

general region wide open channel may be considered as a same as flow in the rectangular channel of infinite width.

27) What do you mean by velocity distribution in an open channel?

The non uniform distribution of velocity in an open channel is due to

- 1) Presence of free surface.
- 2) Frictional resistance along the channel boundary
- 3) The velocity distribution in a channel is measured with help of Pitot tube.

28) What are the applications of specific energy?

The application of specific energy is:

- 1) Analysis of flow through channel transmission.
- 2) Flow over raised channel bottom
- 3) Flow through sluice gate openings.

29) What are the factors affecting Manning's roughness coefficient?

The factors affecting Manning's roughness coefficient:

- 1) surface roughness
- 2) Channel irregularities
- 3) Silting and scouring
- 4) Obstruction
- 5) Size and shape of channel
- 6) Seasonal changes.
- 7) Suspended material and bed load.
- 8) Canal alignment

30) What is top width, wetted area, wetted perimeter, Hydraulic mean radius, hydraulic depth?

Top width is the width of the channel section at the free surface. (The width of the liquid surface exposure to atmospheric pressure). Wetted area is the cross sectional area

of flow section in the channel. Wetted perimeter the cross sectional area of flow section of the channel.

Hydraulic mean radius (R): $R = A/P$

Hydraulic depth (D): $D = A/T$

16 Marks Questions and Answers

1. Find the diameter of a circular sewer pipe which is laid at a slope of 1 in 8000 and carries a discharge of 800 litres/second when flowing half full. Take the value of Manning's $N=0.02$

Given:

Slope of pipe, $I = 1/8000$

Discharge $Q = 800 \text{ lts/s} = 0.8 \text{ m}^3/\text{s}$

Manning's, $N=0.02$

Dia of sewer pipe = D

Depth of Flow, $d = D/2$

$$\text{Area of flow } A = \frac{\pi}{4} D^2 \times \frac{1}{2}$$

$$= \frac{\pi D^2}{8}$$

$$\text{Wetted perimeter} = \frac{\pi D}{2}$$

$$\text{Hydraulic mean depth, } m = A/P = \frac{\frac{\pi D^2}{8}}{\frac{\pi D}{2}} = D/4$$

$$Q = AC \sqrt{mi} = \frac{\pi D^2}{8} \times \left(\frac{1}{N} m^{1/6} \right) \times \sqrt{mi}$$

$$0.8 = \frac{\pi D^2}{8} \times 1/0.02 \text{ m}^{1/6} \times m^{1/2} \times \sqrt{i}$$

$$0.8 = \frac{\pi D^2}{8} \times 1/0.02 \text{ m} \left(\frac{1}{6} + \frac{1}{2} \right) \times \sqrt{\frac{1}{8000}} = \frac{\pi}{8} D^2 \times 1/0.02 \times m^{2/3} \times 0.01118.$$

$$= 0.2195 \times D^2 \times (D/4)^{2/3} = \frac{0.2195}{4^{2/3}} D^{8/3}$$

$$D^{8/3} = \frac{0.8}{0.0871} = 9.1848$$

$$D = (9.1848)^{3/8} = 2.296 \text{ m}$$

2) i) Find the specific energy of flowing water through a rectangular channel of width 5m when the discharge is 10 m³/s and depth of water is 3m.

Width b=5m

Q= 10 m³/s

h= 3m

$$V = Q/\text{area} = 10/b \times h$$

$$= 10/5 \times 3 = 2/3$$

$$E = h + V^2/2g$$

$$E = 3 + \frac{(2/3)^2}{2 \times 9.81} = 3.0226 \text{ m}$$

(ii) Find the critical depth and critical velocity of water flowing through a rectangular channel of width 5m, when discharge is 15m³/s.

b= 5m

Q= 15 m³/s

$$\text{Discharge per unit, } q = Q/b = 15/5 = 3 \text{ m}^2/\text{s}$$

$$\text{Critical depth } h_c = (q^2 / g)^{1/3} = (3^2 / 9.81)^{1/3} = 0.972 \text{ m}$$

$$\begin{aligned} \text{Critical velocity } V_c &= \sqrt{g x h_c} = \sqrt{9.81 \times 0.972} \\ &= 3.088 \text{ m/s} \end{aligned}$$

3) The discharge of water through a rectangular channel of width 8m, is 15 m³/s when depth of flow of water is 1.2 m. Calculate:

- 1) Specific energy of the flowing water
- 2) Critical depth and critical velocity
- 3) Value of minimum specific energy

Given:

$$\begin{aligned} Q &= 15 \text{ m}^3/\text{s} \\ b &= 8 \text{ m} \\ h &= 1.2 \text{ m} \end{aligned}$$

$$\text{Discharge per unit width} = Q/b = 15/8$$

$$\text{Velocity of flow, } V = Q/\text{area} = 15/b \times h = 15/8 \times 1.2 = 1.5625 \text{ m/s}$$

$$(i) \quad \text{Specific energy } E = h + \frac{V^2}{2g} = 1.2 + \frac{(1.5625)^2}{8 \times 9.81} = 1.324 \text{ m}$$

$$(ii) \quad \text{Critical depth } h_c = \left(\frac{q^2}{g} \right)^{1/3} = \left(\frac{1.875^2}{9.81} \right)^{1/3} = 0.71 \text{ m}$$

$$(iii) \quad \text{Critical velocity } V_c = \sqrt{g x h_c} = \sqrt{9.81 \times 0.71} = 2.639 \text{ m/s.}$$

(iv) Minimum Specific energy (E_{\min}):

$$E_{\min} = \frac{3h_c}{2} = \frac{3 \times 0.71}{2} = 1.065 \text{ m}$$

4) i) The specific energy for a 5m wide rectangular channel is to be 4Nm/N. If the rate of flow of water through the channel is 20 m³/s. Determine the alternate depth of flow.

$$b = 5 \text{ m}$$

$$E = 4 \text{ Nm/N} = 4 \text{ m} \qquad E = h + \frac{V^2}{2g} \qquad V = \text{Discharge/area} = Q/b \times h = 20/5 \times h$$

$$Q = 20 \text{ m}^3/\text{s}$$

$$E = 4$$

$$E = h + \frac{V^2}{2g} = h + (4/h)^2 \times 1/2g$$

$$= h + \frac{8}{gxh^2}$$

$$4 = h + \frac{8}{9.81xh^2} = h + \frac{0.8155}{h^2}$$

$$4h^2 = h^3 + 0.8155 \text{ (or) } h^3 - 4h^2 + 0.8155 = 0$$

This is a cubic equation solving by trial (or) error

$$h = 3.93 \text{ m and } 0.48$$

ii) Derive the froude number.

The square root of ratio of inertia force of flowing liquid to gravity force

$$F_e = \sqrt{\frac{F_i}{F_g}}$$

$$F_i = \rho AV^2 = \text{mass} \times \text{acceleration}$$

$$F_g = M g = \rho ADg$$

$$D = \text{wetted area} / \text{Top width of channel} = A/T$$

$$F_e = \sqrt{\frac{\rho AV^2}{\rho ADg}} = \sqrt{\frac{V^2}{Dg}} = \frac{V}{\sqrt{Dg}}$$

5) Condition for maximum discharge for given value of specific energy.

The specific energy at any section of channel is given

$$E = y + \frac{V^2}{2g} ; \quad V = \frac{Q}{A} = \frac{Q}{by}$$

$$E = y + \frac{Q^2}{2b^2 y^2 g}$$

$$E - y = \frac{Q^2}{2b^2 y^2 g}$$

$$Q^2 = (E - y) 2 b^2 y^2 g$$

$$= 2b^2 g [Ey^2 - y^3]$$

$$Q = b \sqrt{2g(Ey^2 - y^3)} \dots\dots\dots (1)$$

For discharge Q to be maximum to expression $Ey^2 - y^3$ should be maximum

Hence differentiate (1) with respect to y and equate it to zero

$$\frac{dQ}{dy} = \frac{b}{2} \left[\left[2g(Ey^2 - y^3) \right]^{\frac{1}{2}} * \left[2g[2Ey - 3y^2] \right] \right]$$

$$= \frac{b}{2} \left[\frac{1}{\sqrt{2g(Ey^2 - y^3)}} * 2g(2Ey - 3y^2) \right]$$

$$\frac{dQ}{dy} = 0$$

$$\frac{bg(2Ey - 3y^2)}{\sqrt{2g(Ey^2 - y^3)}} = 0$$

$$2E bgy - 3 bgy^2 = 0$$

$$E = 3/2 y$$

$$y = 2/3 E$$

Maximum Discharge:

$$Q = b \sqrt{2g\left(\frac{3}{2}y^3 - y^3\right)}$$

$$Q = b \sqrt{gy^3}$$

$$Q = b \sqrt{2g\left(E - \frac{4}{9}E^2 - \frac{8}{27}E^3\right)}$$

$$Q_{\max} = 1.705 b E^{3/2}$$

The specific energy is minimum when it is equal to 3/2 times value of depth of critical flow.

Here the equation (2) represents the specific energy and is equal to 3/2 times the depth of flow. Therefore the equation (2) represents the specific energy and y is the critical depth.

Hence the condition for maximum discharge for given value of specific energy is that the depth of flow should be critical.

6) i) Find the velocity of flow and rate of flow of water through a rectangular channel of 6m wide and 3m deep, when it is running full. The channel is having bed slope as 1 in 2000. Take chezy's constant $C = 55$.

Given:

Width of rectangle channel, $b = 6\text{m}$.

Depth $d = 3\text{m}$

Solution:

Area = $b \times d = 6 \times 3 = 18\text{m}^2$

Bed Slope, $i = 1 \text{ in } 2000 = 1/2000$

Chezy's constant $C = 55$

Perimeter $P = b + 2d = 6 + 2 \times 3 = 12\text{m}$

Hydraulic mean depth, $m = A/P = 18/12 = 1.5\text{m}$

$V = C\sqrt{1.5 \times 1/2000} = 1.506 \text{ m/s}$

$Q = V \times \text{Area} = 1.506 \times 18 = 27.108\text{m}^3/\text{s}$.

ii) Find the slope of the bed of a rectangular channel 5m when depth of water is 2m and rate of flow is given as $20 \text{ m}^3/\text{s}$. Take chezy's constant, $C = 50$.

Given:

Width of channel $b = 5\text{m}$.

Depth of water $d = 2\text{m}$

Rate of flow $Q = 20 \text{ m}^3/\text{s}$.

$C = 50$

Bed Slope = i

Solution:

$Q = AC \sqrt{mi}$

$$A = \text{Area} = b \times d = 5 \times 2 = 10 \text{ m}^2$$

$$m = A/P = 10 / (b+2d) = 10 / (5+2 \times 2) = 10 / (5+4) = 10/9 \text{ m}$$

$$20.0 = 10 \times 50 \times \sqrt{10/9 \times i}$$

$$\sqrt{10/9 \times i} = 20/500 = 2/50 \quad 10i/9 = 4/2500$$

$$i = 4/2500 \times 9/10 = 36/25000 = 1/25000/36 = 1/694.44$$

Therefore Bed slope is 1 in 694.44

7) A flow of water of 100 lts/sec flows down in rectangular flume of width 600mm and having adjustable bottom slope. If chezy's constant C is 56, find the bottom slope necessary for uniform flow with a depth of flow of 300mm. Also find the conveyance K of the flume.

Given:

$$\text{Discharge } Q = 100 \text{ lts/s} = 100/1000 = 0.1 \text{ m}^3/\text{s}.$$

$$b = 600\text{mm} = 0.6\text{m}$$

$$d = 300\text{mm} = 0.3\text{m}$$

$$A = b \times d = 0.6 \times 0.3 = 0.18\text{m}^2$$

$$C = 56$$

$$\text{Slope of bed} = i$$

Solution:

$$\text{Hydraulic mean depth, } m = A/P = 0.18/(b+2d) = 0.18/(0.6+2 \times 0.3) \\ = 0.18/1.2 = 0.15\text{m}$$

$$Q = AC \sqrt{mi}$$

$$0.1 = 0.18 \times 56 \sqrt{0.15 \times i} \implies \sqrt{0.15 \times i} = 0.1/(0.18 \times 56)$$

$$\text{Squaring on both sides } 0.15i = (0.10/0.18 \times 56)^2 = 0.00098418$$

$$i = 0.00098418 / 0.15 = 0.0006512 = 1/1/0.0006512 = 1/1524$$

Therefore Slope of the bed is 1 in 1524.

Conveyance K of the channel

$$Q = AC \sqrt{mi} = Q = K \sqrt{i}$$

$$\text{Where, } K = AC \sqrt{m} \implies \text{Conveyance of channel section}$$

$$K = AC \sqrt{m} = 0.18 \times 56 \times \sqrt{0.15} = \mathbf{3.9039 \text{ m}^3/\text{s}}.$$

8) Find the discharge through a trapezoidal channel of width 8m and side slope of 1 horizontal to 3 vertical. The depth of flow of water is 2.4m and value of Chezy's constant, C = 50. The slope of the bed of the channel is given 1 in 4000.

Given:

$$\text{Width } b = 8\text{m}$$

$$\text{Side Slope} = 1 \text{ horizontal to } 3 \text{ vertical}$$

$$\text{Depth } d = 2.4\text{m}$$

$$\text{Chezy's constant } C = 50, \text{ Bed Slope } I = 1/4000$$

Depth CF = 2.4

Solution:

Horizontal dts BE = $2.4 \times 1/3 = 0.8\text{m}$

Therefore Top Width of the channel,

$CD = AB + 2 \times BE = 8.0 + 2 \times 0.8 = 9.6\text{m}$

Therefore Area of trapezoidal channel, ABCD is given as,

$A = (AB + CD) \times CE/2 = (8+9.6) \times 2.4/2 = 17.6 \times 1.2 = 21.12\text{m}^2$

Wetted Perimeter, $P = AB + BC + AD = AB + 2BC$

$BC = \sqrt{BE^2 + CE^2} = \sqrt{(0.8)^2 + (2.4)^2} = 2.529\text{m}$

$P = 8 + 2 \times 2.529 = 13.058\text{m}$

Hydraulic mean depth $m = A/P = 21.12/13.058 = 1.617\text{m}$

$Q = AC\sqrt{mi} = 21.12 \times 50 \sqrt{1.617 \times 1/4000} = \mathbf{21.23 \text{ m}^3/\text{s}}$.

9) Find the bed slope of trapezoidal channel of bed width 6m, depth of water 3m and side slope of 3 horizontal to 4 vertical, when the discharge through the channel is $30 \text{ m}^3/\text{s}$. Take Chezy's Constant, $C = 70$

Given:

Bed Width, $b = 6.0\text{m}$

Depth of flow, $d = 3.0\text{m}$

Side Slope = 3 Horizontal to 4 vertical

Discharge $Q = 30 \text{ m}^3/\text{s}$

Depth of flow CE = 3m

Chezy's Constant = 70

$CE = 3\text{m}$

$BE = 3 \times \frac{3}{4} = 9/4 = 2.25\text{m}$

Therefore Top Width, $CD = AB + 2 \times BE = 6.0 + 2 \times 2.25 = 10.50\text{m}$

Wetted Perimeter, $P = AD + AB + BC = AB + 2BC$ (... $BC = AD$)

$= AB + 2 \sqrt{BE^2 + CE^2} = 6.0 + 2 \sqrt{(2.25)^2 + (3)^2} = 13.5\text{m}$

$A = \text{Area of trapezoidal ABCD}$

$= (AB+CD) \times CE / 2 = (6+10.50)/2 \times 3 = 24.75\text{m}^2$

Hydraulic mean depth, $m = A/P = 24.75/13.50 = 1.833$

$Q = AC\sqrt{mi}$

$30.0 = 24.75 \times 70 \sqrt{1.833 \times i} = 2345.6 \sqrt{i}$

$i = (30/2345.6)^2 = 1/(2345.6/30)^2 = 1/6133$

$i = 1/6133$

10) i) Find the discharge of water through the channel shown in the fig. Take the value of Chezy's constant = 60 and slope of the bed as 1 in 2000.

Given:

Chezy's Constant $C = 60$

Bed Slope, $i = 1/2000$

Solution:

$$\begin{aligned}
 A &= \text{Area ABCD} + \text{Area BEC} \\
 &= (1.2 \times 3.0) + \pi R^2/2 = 3.6 + (1.5)^2 \pi/2 = 7.134 \text{ m}^2 \\
 \text{Wetted Perimeter, } P &= AB + BEC + CD \\
 &= 1.2 + \pi R + 1.2 = 1.2 + \pi 1.5 + 1.2 \\
 &= 7.1124 \text{ m} \\
 \text{Hydraulic mean depth, } m &= A/P = 7.134/7.1124 = 1.003 \\
 Q &= AC\sqrt{mi} = 7.134 \times 60 \times \sqrt{(1.003 \times 1/2000)} = \mathbf{9.585 \text{ m}^3/\text{s}}
 \end{aligned}$$

ii) Find the rate of flow of water through a V- Shaped channel as shown in the fig. Take the value of C = 55 and slope of the bed 1 in 2000

Given:

$$\begin{aligned}
 C &= 55 \\
 \text{Bed Slope } i &= 1/1000 \\
 \text{Depth of flow, } d &= 4.0 \text{ m} \\
 \text{Angle made by each side with vertical i.e } \angle ABD &= \angle CBD = 30^\circ
 \end{aligned}$$

Solution:

$$\begin{aligned}
 \text{Area A} &= \text{Area of ABC} \\
 &= 2 \times \text{Area of ABCD} = (2 \times AD \times BD)/2 = AD \times BD \\
 &= BD \tan 30^\circ \times BD \\
 &= 4 \tan 30^\circ \times 4 = 9.2376 \text{ m}^2
 \end{aligned}$$

$$\left(\begin{array}{l} \tan 30^\circ = AD/BD \\ AD = BD \tan 30^\circ \\ (AB = BC) \end{array} \right)$$

$$\begin{aligned}
 \text{Wetted Perimeter, } P &= AB + BC = 2AB \\
 &= 2\sqrt{(BD^2 + AD^2)} = 2\sqrt{(4)^2 + (4 \tan 30^\circ)^2} \\
 &= 2\sqrt{(16.0 + 5.333)} = 9.2375 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Therefore Hydraulic mean depth, } m &= A/P = 9.2376/9.2375 = 1.0 \text{ m} \\
 Q &= AC\sqrt{mi} \\
 &= 9.2376 \times 55 \times \sqrt{(1 \times 1/1000)} \\
 &= \mathbf{16.066 \text{ m}^3/\text{s}}
 \end{aligned}$$

Empirical Formula for the value of Chezy's constant:
Derive the dimension of C:

$$\begin{aligned}
 V &= C\sqrt{mi} \implies \text{Chezy's formula} \\
 C &= V/\sqrt{mi} = L/T/\sqrt{(L^2/L^3) i} = L/T/\sqrt{(L^2/L^3)} = L/T/\sqrt{L} = \sqrt{L} \cdot \sqrt{L/T} \sqrt{L} \\
 &= \sqrt{L/T} = L^{1/2} T^{-1} \quad [\dots i \text{ is dimensionless}] \\
 C &= L^{1/2} T^{-1}
 \end{aligned}$$

11) With neat sketches give the computation of critical flow.

Critical flow and its computations:

Critical state of flow in an open channel has the following properties:

1. The specific energy is a minimum for a given discharge.
2. The discharge is a maximum for a given specific energy.
3. The specific force is minimum for a given discharge
4. The velocity head is equal to half the hydraulic depth.
5. The Froude number is unity.
6. The slope of the channel that sustains a discharge of a uniform depth equal to critical depth is called critical slope.
7. A control section is one where a definitive stage (depth) – discharge.
8. A critical flow section is excellent control section because, at that section.

$Q = Q_c = A \text{ square of } gD = by_x \text{ square root of } gy$, if the cross section is rectangular.

Typical cross sections:

12) i) Derive the equation for minimum specific energy in terms of critical depth.

Minimum specific energy in terms of critical depth.

Specific energy is given by

$$E = h + \frac{q^2}{2gh^2} \text{ ----- (1)}$$

The specific energy is minimum when depth of flow is critical and hence the above eqn.

$$E_{\min} = h_c + \frac{q^2}{2gh_c^2} \text{ ----- (2)}$$

$$\text{But } h_c = \left(\frac{q^2}{g} \right)^{\frac{1}{3}} \text{ ----- (3)}$$

Using (3) in (2) we get $E_{\min} = h_c + \frac{hc^3}{2hc^2}$

$$E_{\min} = \frac{3h_c}{2}$$

ii) How do you obtain the specific energy curve explain briefly?

Specific energy curve may be obtained by drawing a curve for specific energy E against depth of flow Y .

Diagram:

Consider a rectangular section in which a steady but non uniform flow is taking place.

B = width of channel

Y = depth of flow

Q = discharge through channel

$$\text{Velocity of flow } V = \frac{Q}{A} = \frac{Q}{bh} = \frac{q}{h}$$

q = discharge per unit width.

$$\text{Specific energy } E = h + \left(\frac{\frac{q}{h}}{2g} \right)^2$$

$$= h + \frac{q^2}{2gh^2}$$

$$E = E_p + E_k$$

The specific energy plot entails the following information:

- 1) The curve for potential energy is $E_p = h$ is a straight line passing through the origin making an angle 45° with each of the two axis (X & Y)
- 2) The curve for kinetic energy (E_k) is a parabola.
- 3) Plot for specific energy is obtained by adding kinetic energy to potential energy.
- 4) Specific energy is asymptotic to the horizontal axis for small values of y and asymptotic to 45° lines for high values of y .
- 5) At a certain depth h_c called critical depth the specific energy curve as a point of minimum specific energy, the corresponding flow velocity is called critical velocity V_c .
- 6) If a line is drawn through the point C, the area above that line is known as the area of sub critical flow ($F_e < 1$). Area below that line is known as super critical flow ($F_e > 1$)
- 7) For energy value of specific energy other than minimum there are two possible depth of flow (h_1 & h_2). One greater than critical depth. & other less than critical depth.

This two depth for the same specific energy are referred to as alternate or conjugate depth.

13) Derive the equation for critical depth.

Critical depth (Y_c)

The depth of flow at which the specific energy is minimum is called critical depth. (y_c).

The depth is said to be critical at any section of F_e is equal to one.

$$F_e = \frac{v}{\sqrt{gD}}$$

The mathematical expression for critical depth can be obtained by differentiating the specific energy equation with respects to and equates it to zero.

$$E = h + \frac{q^2}{2gh^2}$$

$$\frac{de}{dh} = 1 + \left(\frac{-2q^2}{2hg^3} \right) = 0 \text{ ----- (1)}$$

$$1 - \frac{q^2}{gh^3} = 0$$

$$h = \left(\frac{q^2}{g} \right)^{\frac{1}{3}}$$

But specific energy is min, the depth of flow is critical,

Therefore,

$$h_c = \left(\frac{q^2}{g} \right)^{\frac{1}{3}}$$

From eq (1);

$$1 - \frac{q^2}{gh^3} = 0$$

$$1 = \frac{q^2}{gh^3} \text{ ----- (2)}$$

We know that $V = \frac{q}{h}$

Sub $V = \frac{q}{h}$ in eq (2)

$$1 = \frac{v^2}{gh} \text{ ----- (3)}$$

$$V = \sqrt{gh}$$

$$\frac{v}{\sqrt{gh}} = 1 \text{ } F_e = 1 \text{ the depth is known as critical depth.}$$

14) i) Give the application of specific energy and discharge curve.

Applications of specific energy:

Analysis of flow through channel transmission
 Flow over raised channel bottom
 Flow through sluice gate openings.

ii) Discharge curve:

For a constant specific energy the discharge per unit width (q) is plotted against depth of flow, the curve is known as discharge curve; In this fig. q attains max value at a particular value of h .

iii) Uniform flow occurs at a depth of 1.5 m in a long rectangular channel 3m wide and laid at a slope of 0.0009. If Manning's $N = 0.015$. Calculate (1) max height of jump on the flow to produce the critical depth. (2) The width of the contraction which will produce critical depth without increasing the upstream depth of flow.

Given:

$$y = 1.5 \text{ m } i = 0.0009$$

$$b = 3 \text{ m } N = 0.015$$

Soln:

$$Y_c = \left(\frac{q^2}{g} \right)^{\frac{1}{3}}$$

$$R = \frac{bd}{b + 2d} = \frac{1.5 \times 3}{3 + 2 \times 1.5} = 0.75$$

$$C = \frac{1}{N} R^{\frac{1}{6}} = \frac{1}{0.015} (0.75)^{\frac{1}{6}}$$

$$C = 64$$

$$V = C \sqrt{mi}$$

$$= 64 \sqrt{0.75 \times 0.0009}$$

$$V = 1.66 \text{ m/s}$$

$$Q = A \times V = 1.5 \times 3 \times 1.66 = 7.48 \text{ m}^3/\text{s}$$

$$q = Q/b = 7.48 / 3 = 2.493$$

$$y_c = \left(\frac{q^2}{g} \right)^{\frac{1}{3}} = \left(\frac{2.493^2}{9.81} \right)^{\frac{1}{3}}$$

$$y_c = 0.859\text{m}$$

$$\begin{aligned} \Delta z_{\text{Max}} &= y_1 + \frac{V_1^2}{2g} - \frac{3y_c}{2} \\ &= 1.5 + \frac{1.66^2}{2 \times 9.81} - \frac{3 \times 0.859}{2} \\ &= 1.5 + 0.1404 - 1.29 \end{aligned}$$

$$\Delta z_{\text{Max}} = 0.3504\text{m}$$

$$\begin{aligned} y_1 + \frac{v_1^2}{2g} &= Y_c + \frac{V_c^2}{2g} \\ 11.5 + \frac{1.66^2}{2 \times 9.81} &= Y_c + \frac{Y_c}{2} \\ 1.64 &= 3Y_c/2 \end{aligned}$$

$$1.64 = \frac{3}{2} \left[\frac{q^2}{g} \right]^{\frac{1}{3}}$$

$$1.093 = q^{2/3}$$

$$q = 3.5795$$

$$q = Q/b = b = \frac{7.48}{3.5795}$$

$$\mathbf{b = 2.089.}$$

CE2253- APPLIED HYDRAULIC ENGINEERING
(FOR IV – SEMESTER)

UNIT – II- UNIFORM FLOW

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PRINCIPAL

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UNIT- II

UNIFORM FLOW

Uniform flow – Velocity measurement – Manning's & Chezy's formula determination of roughness coefficients – Determination of normal depth and velocity – Most economical sections – Non-erodible channels.

Two Marks Questions and Answers

1. Define uniform flow.

For a given length of channel the velocity of flow, depth of flow, slope of channel the c/s remain constant the flow is said to be uniform flow.

$$\frac{\partial V}{\partial S} = 0, \quad \frac{\partial y}{\partial S} = 0,$$

2. Define channel of most economical sections.

A channel which given maximum discharge for a given cross-sectional area and bed slope is called a channel of most economical gross-section. It can also be defined as the channel that has a minimum wetted perimeter, so that there is a minimum resistance to flow and thus resulting in a maximum discharge.

3. What are the conditions to be most economical section?

The conditions to be most economical for the following shapes of the channels will be considered.

1. Rectangular channel
2. Trapezoidal channel
3. Circular channel

4. Relate discharge with wetted perimeter.

$$Q = AC\sqrt{mi}$$

$$= AC\sqrt{\frac{A}{P} \times i}$$

$$= K \frac{1}{\sqrt{P}}, \text{ Where}$$

$$K = AC\sqrt{A \times i} = \text{Constant}$$

$$m = \frac{A}{P}$$

Q will be maximum when the wetted perimeter P is minimum

5. Give the conditions for a rectangular channel to be most economical.

A rectangular channel to be most economical is:

1.
$$b = 2d$$

2.
$$m = \frac{d}{2}$$

Where,

b = width of the channel
d = depth of the channel
m = hydraulic mean depth.

6. What is the condition for the most economical trapezoidal section?

1.
$$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}$$

Half of top width = one of the sloping side

2.
$$m = \frac{d}{2}$$

3. A semi-circle drawn from O with radius equal to depth of flow will touch the three sides of the channel.

7. Give the formula to find the width and perimeter for a trapezoidal section to be most economical.

i.)
$$b = \frac{2d}{\sqrt{3}}$$

ii.)
$$P = 3 \times \frac{2d}{\sqrt{3}}$$

$$P = 3 \times b$$

For a slope of 60° , the length of sloping side is equal to the width of the trapezoidal section.

8. Give the two conditions for the circular channel to be most economical.

1. Condition for maximum velocity
2. Condition for maximum discharge.

9. Give the condition for maximum velocity and maximum discharge.

1. Condition for maximum velocity for circular section.

$$d = 0.81 D$$

D \longrightarrow diameter of the circular channel.

$$m = 0.3 D$$

m \longrightarrow hydraulic mean depth.

2. Condition for maximum discharge for circular section

$$d = 0.95 D$$

10. What are the factors affecting chezy's and manning's N formula?

1. Surface roughness and vegetation.
2. Irregularity in cross=section.
3. Obstruction to flow
4. Sitting
5. Depth flow and discharge
6. Size & shafe of the channel
7. suspended and bed particles
8. Personal changes which after the fluid viscosity.

11. Give the chezy's formula.

$$V = c\sqrt{mi}$$

$$Q = A \times C\sqrt{mi}$$

Where,

Q \longrightarrow discharge
A \longrightarrow area

m \longrightarrow hydraulic mean depth
C \longrightarrow Chezy's constant

12. Drive the dimension of C

$$V = c\sqrt{mi}$$

$$C = \frac{V}{\sqrt{mi}} = \frac{L/T}{\sqrt{\frac{A}{P}i}} = \frac{L/T}{\sqrt{\frac{L^2}{L}i}} = \frac{L}{T\sqrt{Li}} = \frac{\sqrt{L} \times \sqrt{L}}{T\sqrt{Li}}$$

$$\frac{\sqrt{L}}{T} = L^{1/2}T^{-1} \quad (I \longrightarrow \text{dimension})$$

$$C = L^{1/2}T^{-1}$$

13. Give the Bazin formula.

$$C = \frac{157.6}{181 + \frac{K}{\sqrt{m}}} \quad m \longrightarrow \text{hydraulic mean depth (or) hydraulic radius}$$

K Bazin's constant (depends upon the roughness of the surface of the channel)

14. Represent Kutter's formula in MKS Units

$$C = \frac{23 + \frac{0.00155}{i} + \frac{1}{N}}{1 + \left(23 + \frac{0.00155}{i}\right) \frac{N}{\sqrt{m}}}$$

N \longrightarrow Roughness Co-efficient (or) Kutter's constant
i \longrightarrow Slope of the bed
M \longrightarrow hydraulic mean depth

15. Give the manning's formula

$$C = \frac{1}{N} m^{1/6}$$

m \longrightarrow hydraulic mean depth
N \longrightarrow Manning's constant (same value as Kutter's Constant)

16. What are non- erodible channel?

Most lined channel and built up channel can withstand erosion satisfactorily and they are considered non erodible.

In designing non erodible channel, the factors such as max permissible velocity, maximum tractive force are not to be considered.

17. What are the factors to be considered are?

1. The kind of material forming channel body
2. To determine the roughness co-efficient.
3. The maximum permissible velocity to avoid the deposition of silt

18. Give some non-erodible materials.

The materials are:

- Concrete
- Stone masonry
- Steel
- Cast iron
- Timber
- Glass
- Plastic

19. How do you find mean velocity of flow?

The mean velocity of flow is found by,

1. Pitot tube
2. Floats
3. Current meter.

20. What is current meter?

A current meter is an instrument used to measure the velocity of flow at a required point in the flowing stream. It consists of wheel or revolving element containing blades or cups and tail on which flat vane or fins are fixed.

21. On what the value of chezy's constant C depends?

Its value depends upon the roughness of the inside surface of the channels. If the surface is smooth there will be less frictional resistance to the motion of water. Therefore C will have more value and it leads to velocity, discharge increase.

If the surface is rough- vice versa.

22. Define channels of most economical sections?

A channel which gives maximum discharge for a given cross-sectional area and bed slope is called a channel of most economical cross-section.

It is channel which involves least excavation for a designed amount of discharge.

A Channel that has a maximum wetted perimeter, so that there is a minimum resistance to flow and thus resulting in a maximum discharge.

16 MARKS QUESTIONS AND ANSWERS

1) a) A rectangular channel of width, 4m is having a bed slope of 1 in 1500. Find the maximum discharge through the channel. Take value of C = 50

Given:

$$b = 4 \text{ m} \quad i = \frac{1}{15000} \quad C = 50$$

$$b = 2d \quad (\text{or}) \quad d = \frac{b}{2} = \frac{4}{2} = 2.0 \text{ m}$$

$$m = \frac{d}{2} = \frac{2}{2} = 1.0 \text{ m}$$

Area of economical rectangular channel,

$$A = b \times d = 4 \times 2 = 8 \text{ m}^2$$

$$\begin{aligned} Q &= AC\sqrt{m \times i} = (4 \times 2) \times 50 \times \sqrt{1 \times \frac{1}{1500}} \\ &= 10.328 \text{ m}^3/\text{s}. \end{aligned}$$

(b) A rectangular channel carries water at the rate of 400 lts/s when bed slope is 1 in 2000. Find the most economical dimension of the channel of C = 50

Given:

$$Q = 400 \text{ lts/s} = 0.4 \text{ m}^3/\text{s}, \quad i = \frac{1}{2000}, \quad C = 50$$

For the rectangular channel to be most economical,

i. Width $b = 2d$.

ii. Hydraulic mean depth $m = \frac{d}{2}$

$$Area = b \times d = 2d \times d = 2d^2$$

$$Q = AC\sqrt{mi}$$

$$0.4 = 2d^2 \times 50 \sqrt{\frac{d}{2} \times \frac{1}{2000}} = 2 \times 50 \sqrt{\frac{5}{2 \times 2000}} d^{5/2} = 1.581 d^{5/2}$$

$$d^{5/2} = \frac{0.4}{1.581} = 0.253 \Rightarrow (0.253)^{2/3} = 0.577 \text{ m}$$

$$b = 2d = 2 \times 0.577 = 1.154 \text{ m}$$

2. A rectangular channel 4m has depth of water 1.5 m. The slope of the bed of the channel is 1 in 1000 and value of chezy's constant $C = 55$. It is desired to increase the discharge to a maximum by changing the dimensions of the section for constant area of cross-section, slope of the bed and roughness of the channel. Find the new dimension of the channel and increase in discharge.

Given,

$$b = 4\text{m}, \quad A = b \times d = 4 \times 1.5 = 6.0 \text{ m}^2$$

$$d = 1.5 \text{ m} \quad i = \frac{1}{1000}, \quad C = 55$$

Wetted perimeter, $P = d + b + D = 1.5 + 4 + 1.5 = 7.0\text{m}$

$$m = \frac{A}{P} = \frac{4}{7} = 0.857$$

$$Q = AC\sqrt{mi} = 6.0 \times 55 \sqrt{0.857 \times \frac{1}{1000}} = 9.66 \text{ m}^3 / \text{s}$$

For max discharge for a given area, slope of bed and roughness.

Let

b^1 = new width of channel

d^1 = new depth of flow

Area $A = b^1 \times d^1$, where $A = 6 \text{ m}^2$
 $B = b^1 \times d^1$

Max discharge $b^1 = 2d^1$

$$6 = 2d^1 \times d^1 \Rightarrow d^{1^2} = \frac{6}{2} = 3 \Rightarrow d^1 = \sqrt{3} = 1.732$$

$$b^1 = 2 \times 1.732 = 3.464$$

New dimension $b^1 = 3.464\text{m}$ $d^1 = 1.732\text{m}$

Wetted perimeter $p^1 = d^1 + b^1 + d^1 = 1.732 + 3.464 + 1.732$
 $= 6.928$

Hydraulic mean depth, $m^1 = \frac{A}{P^1} = \frac{6}{6.928} = 0.866\text{m}$

$$\left(m^1 = \frac{d^1}{2} = \frac{1.73}{2} = 0.866n \right)$$

$$\text{Max discharge } Q^1 = AC\sqrt{m^1 i} = 6 \times 55 \times \sqrt{0.866 \times \frac{1}{1000}} = 9.71 \text{ m}^3 / \text{s}.$$

$$\therefore \text{Increase in discharge} = Q^1 - Q = 9.71 - 9.66 = 0.05 \text{ m}^3 / \text{s}$$

3. A trapezoidal channel has side slopes 1 to 1. It is required to discharge 13.75 m³/s of water with a bed gradient of 1 in 1000. If unlined the value of chezy's C is 44. If lined with concrete, its value in 60. The cost per m³ of excavation is four times the cost per m² of lining. The channel is to be the most efficient one find whether the lined canal or the unlined canal will be cheaper. What will be the dimension of hat economical canal?

Given,

$$\text{Side slope } n = \frac{1}{1} = 1$$

$$\text{Slope of bed } i = \frac{1}{1000}$$

$$Q = 13.75 \text{ m}^3 / \text{s}$$

$$\text{For unlined } \longrightarrow C = 44$$

$$\text{Lined } \longrightarrow C = 60$$

Cost per m³ of excavation = 4 x cost per m² of lining.

$$\text{Let the cost per m}^2 \text{ of lining} = x$$

$$\text{Cost per m}^3 \text{ of excavation} = 4x.$$

For most efficient trapezoidal channel, Hydraulic mean depth $i = m = \frac{d}{2}$

d \longrightarrow depth of channel

b \longrightarrow width of channel

2. Half of top width = length of sloping side

$$\boxed{\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}}$$

$$\frac{b + 2 \times 1 \times d}{2} = d\sqrt{1^2 + 1} = d\sqrt{2}$$

$$b = 2 \times \sqrt{2}.d - 2d = 0.828d$$

$$b = 0.828d$$

$$A = (b + nd) \times d = (0.828d + 1 \times d) \times d$$

$$A = 1.828d^2$$

1. For unlined channel:

$$C = 44$$

$$Q = A \times V = A \times C \sqrt{mi}$$

$$Q = AC \sqrt{mi}$$

$$13.75 = 1.828d^2 \times 44 \times \sqrt{\frac{d}{2} \times \frac{1}{1000}}$$

$$\left(\because A = 1.828d^2, m = \frac{d}{2} \right)$$

$$= \frac{1.828 \times 44}{\sqrt{2000}} \times d^{5/2}$$

$$d^{5/2} = \frac{13.75 \sqrt{2000}}{1.828 \times 44} = 7.452 \Rightarrow d = (7.6452)^{2/5} = 2.256m.$$

$$d = 2.256m. \text{ Subs in (1) we get,}$$

$$b = 0.828 \quad d = 0.828 \times 2.256 = 1.868m$$

Cost of excavation per running meter

$$\frac{\text{Length of unlined channel}}{\text{Cost per m}^3 \text{ of excavation.}} = \frac{\text{Volume of channel} \times \text{Cost per m}^3 \text{ of excavation.}}{\text{Cost per m}^3 \text{ of excavation.}}$$

$$= (\text{Area of channel} \times 1) \times 4x = [(b + nd) \times d \times 1] \times 4x$$

$$= (1.868 + 1 \times 2.256) \times 2.256 \times 1 \times 4x = 37.215x$$

2. For lined channels

$$\text{Value of } C = 60$$

$$Q = A \times C \times \sqrt{m \times i}$$

Subs the value of A from equn (2) and $m = \frac{d}{2}$

$$13.75 = 1.828d^2 \times 60 \times \sqrt{\frac{d}{2} \times \frac{1}{1000}} \quad (\because Q = 13.75)$$

$$= 1.828d \times 60 \times \frac{1}{\sqrt{2000}} \times d^{5/2}$$

$$d^{5/2} = \frac{13.75 \times \sqrt{2000}}{1.828 \times 60} = 5.606$$

$d = 1.992\text{m}$

subs in (1) $b = 0.828 d = 0.828 \times 1.992 = 1.649 \text{ m}$

\nearrow The cost of lining
 In the case of lined channel
 \searrow
 Cost of excavation

Cost of excavation = Volume of channel \times cost per m^3 of excavation.

$$= \left[(b + nd)^4 \times d \right]^{x/l} \times 1 \times 4x$$

$$(1.649 + 1 \times 1.992) \times 1.992 \times 1 \times 4x = 29.01x$$

Cost of lining =	Area of lining	x	cost per m^2 of lining
	P	x	1
	= (Perimeter of lining \times 1)	x	x
	$= (b + 2d\sqrt{1+n^2}) \times 1 \times x$		
	$= (1.649 + 2 \times 1.992\sqrt{1+1^2}) \times 1 \times x$		
	$= (1.649 + 2 \times 1.992\sqrt{2}) x = 7.283x$		

$$\text{Total cost} = 29.01x + 7.283x = 36.293x$$

$$\begin{aligned}\text{The total cost of lined channel} &= 36.293x \\ \text{Unlined channel} &= 37.215x.\end{aligned}$$

Hence \longrightarrow Lined channel will be cheaper.

Dimensions

$$b = 1.649 \text{ m}$$

$$d = 1.992 \text{ m}$$

4. A power canal of trapezoidal section has to be excavated through hard clay at the least cost. Determine the dimensions of the channel given, discharge equal to $14 \text{ m}^3/\text{s}$ bed slope 1:2500 and Manning's $N = 0.02$

Given:

$$Q = 14 \text{ m}^3/\text{s} \quad N = 0.02$$

$$i = \frac{1}{2500}$$

The trapezoidal section should be most economical for the excavation of the canal at the least cost.

Side slope (Value of n) is not given. Hence the best side slope for most economical trapezoidal section is given by equation.

$$n = \frac{1}{\sqrt{3}}$$

For most economical section,

Half of top width = Length of one of sloping side

$$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}$$

For

$$n = \frac{1}{\sqrt{3}}$$

$$\frac{b + 2 \times \frac{1}{\sqrt{3}} d}{2} = d \sqrt{\left(\frac{1}{3}\right)^2 + 1} = \frac{2d}{\sqrt{3}} \quad \left[\begin{array}{l} b = \frac{2d}{\sqrt{3}} \\ n = \frac{1}{\sqrt{3}} \end{array} \right]$$

Area of trapezoidal section, $A = (b + nd) \times d = \left(\frac{2d}{\sqrt{3}} + \frac{1}{\sqrt{3}} d \right) \times d$

$$A = \sqrt{3} d^2$$

Hydraulic mean depth for most economical section,

$$m = \frac{d}{2}$$

$$Q = AC\sqrt{mi}$$

where

$$C = \frac{1}{N} m^{1/6}$$

$$Q = \sqrt{3} d^2 \times \frac{1}{N} m^{1/6} \sqrt{m \times \frac{1}{2500}}$$

$$= \sqrt{3} d^2 \times \frac{1}{0.02} \times m^{1/6 + 1/2} \sqrt{\frac{1}{2500}} = 1.732 d^2 \times m^{2/3}$$

$$14.0 = 1.732 d^2 \times \left(\frac{d}{2} \right)^{2/3} = \frac{1.732}{2^{2/3}} d^{8/3} = 1.09 d^{8/3}$$

$$d^{8/3} = \frac{14.0}{1.09} = 12.844$$

$$d = (12.844)^{3/8} = (12.844)^{0.375} = 2.605 \text{ m}$$

$$b = \frac{2d}{\sqrt{3}} = \frac{2 \times 2.605}{1.732} = 3.008 \text{ m}$$

5. A trapezoidal channel with side slope of 1 to 1 is to be designed to convey $10\text{m}^3/\text{s}$ at a velocity of 2m/s . So that the amount of concrete lined for bed side is minimum Calculate the area of lining require for 1m length of channel.

$$n = 1 \quad V = 2\text{m/s}$$

$$Q = 10\text{m}^3/\text{s}. \quad Q = A \times V$$

$$A = \frac{Q}{V} = \frac{10}{2} = 5\text{m}^2$$

$$n = \frac{1}{1}$$

$$b + 2nd = 2d\sqrt{1 + n^2}$$

$$b + 2d = 2d\sqrt{2}$$

$$\frac{b + 2d}{2} = g(2.828) \Rightarrow b = 0.828d.$$

$$A = (b + nd)d$$

$$5 = (0.828d + d)d$$

$$2.735 = d^2$$

$$\Rightarrow d = 1.654\text{m}$$

$$b = 1.369\text{m}.$$

Area for 1m length

$$A_{1\text{ m}} = \text{Wetted perimeter} \times \text{length} = (b + 2d) \times \sqrt{1 + n^2} \times 1$$

$$= [1.369 + 2 \times 1.654] \times \sqrt{1 + 1}$$

$$A_{1\text{m}} = 6.61 \text{ m}^2$$

6. What are the factors to be considered for non erodible channels give some examples and explain how to determine the coefficient?

- * Most lined channels and build up channels can with stand erosion satisfactorily and they are consider non erodible.
- * In designing non-erodible channel the factors such as max permissible velocity, max tractive force are not be considered.
- * The designer simply compute the dimension of channel by a uniform flow and the decides the final dimension on the basics of hydraulic efficiency or empirical rule of best section practically and economically.

The factors to be considered are,

- * The kind of material forming channel body
- * To determine the roughness co-efficient
- * The minimum permissible velocity to avoid the deposition of silt and depers.
- * Channel bottom and side slope free board etc all forms the most efficient section.

Some on-erodible:

Concrete	timber
Stone masonry	Class
Steel	Plastic
Cast iron	

The selection of the material depends mainly on the availability of

- Cost of the material.
- Method of construction.
- Purpose for which the channel to be used.

Determination of Manning Roughness Co-efficient:

For the determination of roughness co-efficient N is so difficult for that there is no exact method of selecting n value.

The experienced engineer can calculate by means sound engineering judgment and experience.

For beginners it can be no more than guess and different individual will \obtain different results.

Approaches for Determination of N:

1. To understand factor that effect the value of N and narrow the problem by guess work.
2. To construct a table typical N values for channels of various types.
3. To examine and become familiar with appearance of some typical channel whose roughness co-efficient are known.
4. To determine value of N by analytical procedure based on the theoretical velocity distribution in the channel C/s and on the data of either velocity or roughness co-efficient.

7. Briefly explain the measurement of flow of irregular channel?

The term irregular channel includes large river and small streams.

In case of small streams flow can be obtained by filling notch or weir across the stream and it is not possible in case of large rivers.

Increase of large rivers, discharge is equal to

Area of flow x mean velocity of flow

- Simple segment method.
- Simpson's rule.

Simple segment method:

In this method, the C/s of river is divided into number of segments AB, BC, CD etc as shown in fig.

C/s of river with unequal segments.

$l_1, l_2, l_3, \dots \longrightarrow$ Length of the segment AB, BC, CD...

And $d_1, d_2, d_3, \dots \longrightarrow$ mean depth of respective segments.

\therefore Area of flow = area of segment AB = Area of segment BC +

$$= l_1 b_1 + l_2 b_2 + l_3 b_3 + \dots$$

Simpson's Rule:

In this method the whole river width is divided into even number of equal segments, so that there are odd number of depths taken at an end of each segment as shown in fig.

$$A = \frac{l}{3} (d_0 + d_{lost}) + 2(d_1 + d_3 + d_5) + 4(d_2 + d_4 + d_6)$$

l = length of each segment.

$d_1, d_2 \longrightarrow$ depth taken at the end of segment.

Mean Velocity of flow

- Pitot tube

- Floats \longrightarrow Single float
- \longrightarrow Double float
- \longrightarrow Rod float.

- Current meter.

Pitot Tube:

A pitot tube is a simple device used for measuring the velocity of flow at the required pt in the flowing stream.

It consists of a glass tube bent at right angles

The tube is dipped vertically I the flowing stream with its lower open end facing direction of flow, upper open end projecting above the water level in the stream.

The water rises up in the tube due to pressure exerted y the flowing water.

By measuring rise of water in table. The velocity of water V calculated by,

$$V = \sqrt{2gh}$$

h —————> heat of water in the tube aove the water surface

g —————> acceleration due gravity.

Floats:

A float is a small object made of wood or other suitable material which is lighter than the water and thus capable of floating on surface.

The surface velocity at any section may be obtained by single float.

The time taken by the float to traverse a known distance is measured.

$$\text{Surface velocity (V}_s\text{)} = \frac{\text{Distance traveled by float}}{\text{Time taken to travel the distance}}$$

$$\text{Mean velocity of flow} = 0.8 \text{ to } 0.95 V_s$$

Double Float:

A double float consists of a surface float on which it is attached with a hollow metal sphere heavier than water and suspended from it by a chord of known length.

1. The depth of lower float may be regulated by the length of chord.
2. The velocity is obtained by noting the time taken by the float to traverse a known distance.
3. Double float directly gives the value of mean velocity.

Rod Float:

It consists of vertical wooden rod which is weighted at bottom to keep it vertical.

The length of rod is so adjusted that it reaches bottom of the stream. The rod will travel with a velocity equal to the mean velocity of the section.

Current Meter:

A current meter is an instrument used to measure the velocity of flow at a required pt in the flowing stream. It consists of wheel or revolving element containing blades or cups and tail on which flat vans or fins are fixed.

The current meter according to revolving element may be classified into

1. Cup type
2. Screw type
3. Propeller type

CUP TYPE:

Series of conical cup mounted on a spindle, the spindle held vertical at right angle to direction of flow

SCREW TYPE:

The revolving element consists of shaft with its axis parallel to the direction of flow which carries a number of curved vanes mounted on periphery of shaft.

In order to measure the velocity of flow water submerged under water and motion of water in the stream activate it driving the wheel at a speed proportional to the velocity of flow.

An electric current is passed from the battery to the wheel by means of wire.

The rotation of wheel makes and breaks the electric circuit which causes an electric bell to ring.

Thus by counting the ringing bell the rotation of wheel and hence the velocity of flowing water is calculated.

8. A trapezoidal channel has side slopes of 1 horizontal to 2 vertical and the slope of the bed is 1 in 1500. The area of the section is 40 m². Find the dimensions of the section if it is more economical. Determine the discharge of the most economical Xⁿ if C = 50

$$\text{Side slope, } n = \frac{\text{Horizontal}}{\text{Vertical}} = \frac{1}{2}$$

$$\text{Bed slope, } i = \frac{1}{15000}, \quad C = 50$$

Area of section, $A = 40 \text{ m}^2$

For the most economical section,

$$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1} \quad (\text{or}) \quad \frac{b + 2 \times \frac{1}{2}d}{2} = d\sqrt{\left(\frac{1}{2}\right)^2 + 1}$$

$$\frac{b + d}{2} = d\sqrt{\frac{1}{4} + 1} = 1.118d$$

$$b = 2 \times 1.118d - d = 1.236d$$

Area of trapezoidal section,

$$\boxed{A = \frac{b + (b + 2nd)}{2} \times d} = (b + nd)d$$

$$A = \left(1.236d + \frac{12}{2}d\right)d = 1.736d^2$$

$$40 = 1.736d^2 \Rightarrow d = \sqrt{\frac{40}{1.736}} = 4.80m$$

$$b = 1.236 \times d = 1.236 \times 4.80 = 5.933m$$

Discharge for most economical Xⁿ $m = \frac{d}{2} = \frac{4.80}{2} = 2.40m$

$$Q = AC\sqrt{m \times i} = 40 \times 50 \sqrt{2.4 \times \frac{1}{1500}}$$

$$Q = 80m^3 / s$$

9. A trapezoidal channel has side slopes of 3 horizontal to 4 vertical and slope of its bed is 1 in 2000. Determine the optimum dimensions of the channel, if it is to carry water at 0.5 m³/s. Take chezy's constant 80.

Given,

$$n = \frac{\text{Horizontal}}{\text{Vertical}} = \frac{3}{4}, \quad i = \frac{1}{2000}$$

$$Q = 0.5 \text{ m}^3/\text{s}. \quad C = 80$$

The condition for most economical section,

$$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}$$

$$\frac{b + 2 \times \frac{3}{4}d}{2} = d\sqrt{\left(\frac{3}{4}\right)^2 + 1} = \frac{5}{4}d$$

$$\frac{b + 1.5d}{2} = 1.25d$$

$$b = 2 \times 1.25d - 1.5d = d$$

$b = d$

For the discharge, $Q_z = AC\sqrt{mi}$, $m = \frac{d}{2}$ (most eco Xⁿ)

$$0.5 = A \times 80 \sqrt{\frac{d}{2} \times \frac{1}{2000}}$$

Area of trapezoidal Xⁿ,

$A = (b + nd) \times d$

$$= \left[d + \frac{3}{4}d \right] \times d = \frac{7}{4}d^2 = 1.75d^2$$

$$0.5 = 1.75d^2 \times 80 \times \sqrt{\frac{d}{2} \times \frac{1}{2000}}$$

$$= 2.2135 d^{5/2}$$

$$d = \left(\frac{0.5}{2.2135} \right) = 0.55m$$

$$b = d = 0.55 \text{ m}$$

Optimum dimensions of the channel are width = depth = 0.55m.

10. A trapezoidal channel with side slopes of 1 to 1 has to be designed to convey 10 m³/s at a velocity of 2m/s so that the amount of concrete lining for the bed and sides is the minimum. Calculate the area of lining required for one meter length of canal.

Given:

$$n = \frac{\text{Horizontal}}{\text{Vertical}} = 1$$

Side slope

$$Q = 10 \text{ m}^3/\text{s}, \quad \text{Area} = \frac{\text{Discharge}}{\text{Velocity}} = \frac{10}{2} = 5 \text{ m}^2$$

$$V = 2 \text{ m/s}$$

For most economical trapezoidal section,

Half of the top width = one of the sloping side.

$$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}$$

For $n = 1$, the condition becomes

$$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}$$

$$n=1 \quad \frac{b + 2 \times 1d}{2} = d\sqrt{n^2 + 1} = 1.414d$$

$$A = (b + nd)d = (0.828d + 1 \times d)d$$

$$= 1.828 d^2$$

$$A = 5 \text{ m}^2$$

$$5 = 1.828d^2 \Rightarrow d = \sqrt{\frac{5}{1.828}} = 1.6538 = 1.654 \text{ m}$$

$$b = 0.828d = 0.828 \times 1.654 = 1.369 \text{ m}$$

Area of lining required for one meter length of canal = Wetted perimeter x length of canal

$$= P \times 1$$

$$P = b + 2d\sqrt{n^2 + 1} = 1.369 + 2 \times 1.654\sqrt{1^2 + 1} = 6.047 \text{ m}$$

$$\text{Area of lining} = 6.047 \times 1 = 6.047 \text{ m}^2$$

CE2253- APPLIED HYDRAULIC ENGINEERING
(FOR IV – SEMESTER)

UNIT – III
VARIED FLOW

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UNIT – III

VARIED FLOW

Dynamic equations of gradually varied flow-assumptions- Characteristics of flow profiles - Draw down and back water curves – Profile determination – Graphical integration, direct step and standard step method – flow through transitions.

Two marks Questions and Answers

1. What is gradually varied flow?

If the depth of flow in a channel changes gradually over a long length of the channel the flow is said to be gradually varied flow and is denoted by G.V.F.

2. What are the assumptions made while deriving an equation for G.V.F?

1. The bed slope of the channel is small
2. The flow is steady and hence discharge Q is constant
3. Accelerative effect is negligible and hence hydrostatic pressure distribution prevails over channel cross-section.
4. The energy correction factor, α is unity, $\alpha = 1$
5. The roughness Co-efficient is constant for the length of the channel and it does not depend on the depth of flow.
6. The formula such as Chezy's formula, Manning's formula, which are applicable to the uniform flow are also applicable to the GVF gradually varied flow for determining the slope of energy line.
7. The channel is prismatic

3. Find the rate of change of depth of water in a rectangular channel of 10m wide and 3m deep when the water is flowing with a velocity of 1m/s. The flow of water through the channel of bed slope 1 in 40100, is regulated in such a way that energy line is having a slope of 0.00004.

Given,

$$b = 10 \text{ m} \quad V = 1 \text{ m/s} \quad \text{Bed slope, } i_b = \frac{1}{4000} = 0.00025$$

$$h = 3 \text{ m} \quad \text{Slope of energy line, } i_e = 0.00004$$

$$\text{let the rate of change of depth of water} = \frac{dh}{dx}$$

$$\frac{dh}{dx} = \frac{(i_b - i_e)}{\left(1 - \frac{V^2}{gh}\right)} = \frac{0.00025 - 0.00004}{\left(1 - \frac{1 \times 1}{9.81 \times 3}\right)} = \frac{0.00021}{0.966} = 0.000214$$

4. Sketch back water curve and afflux?

Consider the flow over a dam. On the upstream side of the dam, the depth of water will be rising. If there had not been any obstruction (such as dam) in the path of flow of water in the channel, the depth of water would have been constant as shown by dotted line parallel to the bed of the channel. Due to obstruction, the water level rises and it has maximum depth from the bed at some section.

5. Define Afflux.

Afflux is defined as the maximum increase in water level due to obstruction in the path of flow of water.

$$\boxed{\text{Afflux} = (h_2 - h_1)}$$

h_1 \longrightarrow depth of water at the point, where water starts rising up.

h_2 \longrightarrow maximum height of rising water from bed.

6. What is back water curve and length of back water curve?

The profile of the rising water on the upstream side of the dam is called back water curve. The distance along the bed of the channel between the section where water starts rising to the section where water is having maximum height is known as length of back water curve.

7. What are the basic equations of GVF?

$$\boxed{V = \frac{1}{n} R^{2/3} S_f^{1/2}}$$

$$\boxed{S_f = \frac{n^2 V^2}{R^{4/3}} = \frac{n^2 Q^2}{A^2 R^{4/3}}}$$

Water surface slope $\longrightarrow S_o$ water surface

Bed slope $\longrightarrow S_o$ - bed

The basic differential equation governing the gradually varied follow is

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \frac{Q^2 T}{gA^3}}$$

Written in terms of (e) Specific energy

$$\frac{dE}{dx} = S_o - S_f$$

y \longrightarrow actual depth

y_o \longrightarrow normal depth

y_c \longrightarrow Critical depth

8. What are the classifications of channel flow?

CATEGORIES OF CHANNEL				
S.No	Channel category	Symbol	Characteristic condition	Remarks
1	Mild slope	M	$h_A > h_C$ critical	Sub critical flow at normal depth.
2	Steep slope	S	$H_c > h_A$	Super critical flow at normal depth
3	Critical slope	C	$h_c = h_A$	Critical flow at normal depth
4	Horizontal bed	H	$S_o = 0$ $Y_o = \infty$	Cannot sustain uniform flow
5	Adverse slope	A	$S_o < 0$	Cannot sustain uniform flow

9. What are the classifications of GVF profiles?

Classification of GVF profiles

Channel	Condition	Type
Mild slope	$h > h_n > h_c$	M ₁
	$h_n > h > h_c$	M ₂
	$h_n > h_c > h$	M ₃
Steep slope	$h > h_c > h_n$	S ₁
	$h_c > h > h_n$	S ₂
	$h_c > h_n > h$	S ₃
Critical slope	$h > (h_c = h_n)$	C ₁
	$h < (h_c = h_n)$	C ₂
	$h_c > h$	
Horizontal bed	$h > h_c$	H ₂
	$h < h_c$	H ₃
Adverse slope	$h > h_c$	A ₂
	$h < h_c$	A ₃

10. In a rectangular channel 12 m wide, depth 3.6 m with a velocity of 12m/s. The bed slope of channel is 1 in 4000. If the flow of water through the channel is regulated in such away the energy line having a slope of 0.00004. Find the rate of change of depth of water in the channel.

Given,

$$b = 12\text{m}, h = 3.6\text{m} \quad V = 1.2 \text{ m/s}, \quad i_b = \frac{1}{4000}, \quad i_e = 0.00004$$

$$\frac{dh}{dx} = \frac{i_b - i_e}{1 - \frac{V^2}{gh}} \quad (\text{or}) \quad \frac{i_b - i_e}{1 - F_e^2} = 2.189 \times 10^{-4}$$

11. What is the Critical slope?

The channel bottom is said as critical when the bottom slopes (S_o) is equal to the critical slope S_c .

$$S_o = S_c$$

In this case, normal depth of flow will be equal to the critical depth

$$h_n = h_c$$

12. What is Mild slope?

The channel bottom is said as mild when the bottom slope S_o is less than critical slope S_c .

In this case the normal depth of flow is greater than critical depth

$$S_o < S_c \quad h_n > h_c$$

13. What is Steep slope?

Channel bottom is said as steep when the bottom slope (S_o) is greater than critical slope S_c .

$$S_o > S_c$$

In this case, the normal depth of flow will be less than the critical depth

$$(h_n < h_c)$$

14. Define Horizontal slope.

The channel bottom is said as horizontal when the bottom slope (S_o) is zero.

$$S_o = 0$$

In this case the normal dept of flow will be infinity

$$h_n = \infty$$

15. Define adverse slope.

If the bottom slope is less than 0, then it is called adverse slope

$$S_o < 0$$

In this case the normal dept will be imaginary

16. How do you classify the water curves?

The water curves (i.e. profiles) may be broadly classified into the following five types:

- | | | | |
|----|-------------------------|-------|--|
| 1. | Mild slope curves | | M ₁ , M ₂ , M ₃ |
| 2. | Steep slope curves | | S ₁ , S ₂ , S ₃ |
| 3. | Critical slope curves | | C ₁ , C ₂ , C ₃ |
| 4. | Horizontal slope curves | | H ₁ , H ₂ , H ₃ |
| 5. | Adverse slope curves | | A ₁ , A ₂ , A ₃ |

17. What are the points to be remembered while studying the flow profiles?

1. The flow profiles approach the normal depth line tangentially. But there is an exception for the profiles on critical slopes.
2. The flow profiles approach the critical depth line perpendicularly. But there is an exception for the profiles on critical slopes.
3. All the profiles in zone 1 and 3 are backwater curve.
4. All the profiles in zone 2 are drawn down curves.
5. All the profiles in zone 3 commence from the bed of the channel.
6. The profiles C₁ and C₂ are practically horizontal.

18. What are the three zones of channel bed?

Channel bed slopes and flow profiles:

h_0 \longrightarrow normal depth of flow

h \longrightarrow actual depth of flow

The above figure shows the channel bed line, the critical bed line, normal depth line.

The space above the channel bed is divided into the following 3 zones.

Zone 1 : This is the space above both critical and normal.

Zone 2 : This is the space between NDL and CDL

Zone 3 : This is the space below both the NDL & CDL

Based on the relative values of y_0 & y_c , channel bed slopes are classified as follows.

19. What are the three methods to calculate Surface profiles in prismatic channel?

The three methods to calculate surface profiles are:

- i. Direct step method.
- ii. Standard step method
- iii. Graphical integration method.

20. Give the formulas related to Direct Step Method

There are a host of methods for computing the GVF profiles. The direct step method is a simple procedure suitable for use in prismatic channels. Their basic equation.

$$\boxed{\frac{dE}{dx} = S_0 - S_f}$$

$S_f \longrightarrow$ frictional slope

Is written in finite difference form as,

$$\frac{\Delta E}{\Delta x} = S_0 - \overline{S_f}$$

$$\Delta x = \frac{\Delta E}{S_0 - \overline{S_f}}$$

$$\overline{S_f} = \frac{S_{f1} + S_{f2}}{2}$$

= average friction slope of the reach

16 Marks Questions and Answers

1. Derive the equation of gradually varied flow (or) Non-uniform flow slope of free water surface?

Consider a rectangular channel having gradually varied flow. The depth of flow is gradually decreasing in the direction of flow.

Z	————→	height of bottom of channel above datum
h	————→	depth of flow
V	————→	Mean velocity of flow
i _b	————→	slope of the channel bed.
i _e	————→	slope of the energy line
b	————→	width of channel
Q	————→	discharge through the channel

The energy equation at any section is given by Bernoulli's equation.

$$E = Z + h + \frac{V^2}{2g}$$

Differentiating this equation with respect to x is where x is measured along the bottom of the channel in the direction of flow, we get.

Equation of Non-uniform flow (Slope of free water surface)

$$\frac{dE}{dx} = \frac{dz}{dx} + \frac{dh}{dx} + \frac{d}{dx} \left(\frac{V^2}{2g} \right) \quad (2)$$

Now

$$\frac{d}{dx} \left(\frac{V^2}{2g} \right) = \frac{d}{dx} \frac{d}{dx} \left(\frac{Q^2}{A^2 \times 2g} \right) \quad \left(\because V = \frac{Q}{A} = \frac{Q}{b \times h} \right)$$

(as $A = b \times h$)

$$\frac{d}{dx} \left(\frac{Q^2}{b^2 h^2 \times 2g} \right) = \frac{Q^2}{b^2 \times 2g} \frac{d}{dx} \left(\frac{1}{h^2} \right) \quad (\because Q, b \text{ \& } g \text{ are constants})$$

$$= \frac{Q^2}{b^2 \times 2g} \frac{d}{dh} \left(\frac{1}{h^2} \right) \frac{dh}{dx} = \frac{Q^2}{b^2 \times 2g} \left[\frac{-2}{h^3} \right] \frac{dh}{dx}$$

$$\frac{d}{dx} \left(\frac{V^2}{2g} \right) = \frac{-2Q^2}{b^2 \times 2gh^3} \frac{dh}{dx} = \frac{Q^2}{b^2 h^2 \times gh} \frac{dh}{dx} = -\frac{V^2}{gh} \frac{dh}{dx}$$

Substitute the value of $\frac{d}{dx} \left(\frac{V^2}{2g} \right)$ in equation (ii), we get

$$\frac{dE}{dx} = \frac{dz}{dx} + \frac{dh}{dx} - \frac{V^2}{gh} \frac{dh}{dx} = \frac{dz}{dx} + \frac{dh}{dx} \left[1 - \frac{V^2}{gh} \right] \quad (3)$$

$$\frac{dE}{dx} = \text{Slope of the energy line} = -i_e.$$

$$\frac{dz}{dx} = \text{Slope of the bed of the channel} = -i_b.$$

-Ve sign with i_e & i_b us taken with the increase of x , the value of E and Z decreases.

Substituting the value of $\frac{dE}{dx}$ and $\frac{dz}{dx}$ in equation (3) we get

$$-i_e = -i_b + \frac{dh}{dx} \left[1 - \frac{V^2}{gh} \right] \quad (\text{or}) \quad i_b - i_e = \frac{dh}{dx} \left[1 - \frac{V^2}{gh} \right]$$

$$\frac{dh}{dx} = \frac{(i_b - i_e)}{\left[1 - \frac{V^2}{gh}\right]}$$

$$F_e = \frac{V}{\sqrt{gh}}$$

$$\boxed{\frac{dh}{dx} = \frac{(i_b - i_e)}{\left[1 - (F_e)^2\right]}}$$

$$F_e^2 = \frac{V^2}{gh}$$

$$\left[\because \frac{V}{\sqrt{gh}} = F_e \right]$$

As h is the depth of flow and x is the distance measured along the bottom of the channel hence $\frac{dh}{dx}$ represents the variation of the water depth along the bottom of the channel.

This is also called the slope of the free water surface. Thus

i. When $\boxed{\frac{dh}{dx} = 0}$ h is constant or depth of the water above

the bottom of channel is constant. It means that free surface of water is parallel to the bed of the channel.

ii. When $\frac{dh}{dx} > 0$

Or $\frac{dh}{dx}$ is +ve, it means the depth of water increases in the direction of flow.

The profile of the water so obtained is called **back water curve**.

iii. When $\boxed{\frac{dh}{dx} < 0}$ or $\frac{dh}{dx}$ is -ve, it means the depth of

Water increases in the direction of flow. The profile of the water is obtained is called Drop down Curve.

2. Find the slope of the free water surface in a rectangular channel of width 20m having depth of flow 5m. The discharge through the channel is 50 m³/s. The bed of the channel is having a slope of 1 in 4000. Take the value of Chezy's constant C = 60

Given,

$$b = 20 \text{ m} \quad \text{bed slope } i_b = \frac{1}{4000} = 0.00025$$

$$h = 5 \text{ m}$$

$$Q = 50 \text{ m}^3 / \text{s} \quad C = 60$$

$$Q = A \times V \Rightarrow \boxed{Q = A \times C \sqrt{mi}} \quad C\sqrt{mi} \quad \text{slope of energy line}$$

$$\text{Area of A flow} = b \times h = 20 \times 5 = 100 \text{ m}^2$$

$$m = \text{hydraulic mean depth} \quad \boxed{m = \frac{A}{P}} = \frac{100}{b+2h} = \frac{100}{20+2 \times 5} = \frac{100}{30}$$

$$m = \frac{10}{3} \text{ m}, i = i_e = \text{Slope of energy line}$$

$$Q = AC \times \sqrt{mi}$$

$$50 = 100 \times 60 \times \sqrt{\frac{10}{3} \times i_e} = 10954.45 \sqrt{i_e}$$

$$i_e = \left(\frac{50}{10954.45} \right)^2 = 0.0000208$$

$$\text{The slope of free water surface} = \frac{dh}{dx}$$

$$\boxed{\frac{dh}{dx} = \frac{i_b - i_e}{1 - \frac{V^2}{gh}}} = \frac{0.00025 - 0.0000208}{1 - \frac{V^2}{9.81 \times 5.0}}$$

$$\boxed{V = \frac{Q}{\text{Area}}} = \frac{50}{b \times h} = \frac{50}{20 \times 5} = 0.5$$

$$\frac{dh}{dx} = \frac{0.00025 - 0.0000208}{1 - \frac{0.5 \times 0.5}{9.81 \times 5.0}} = 0.00023$$

3. Give the expression of the length of Back water curve?

Expression for the length of Back water curve

Consider the flow of water through a channel in which depth of water is rising. Let the two section 1-1 and 2-2 are at such a distance that the distance between them represents the length back water curve.

h_1 \longrightarrow dept of flow at X^n 1-1

V \longrightarrow Velocity of flow at X^n 1-1

V_2 dept of flow at X^n 2-2

i_b \longrightarrow bed slope

L \longrightarrow length of back water curve

Applying Bernoulli's equation at X^n 1-1 and 2-2,

Where, h_L = Loss of energy due to friction $h_L = i_e * L$.

$$Z_1 + h_1 + \frac{V_1^2}{2g} = Z_2 + h_2 + \frac{V_2^2}{2g} + h_L$$

Also taking datum line passing through the bed of the channel at section 2-2. Then $Z_2 = 0$

$$h_L = (i_e \times L) \quad Z_1 = (i_b \times L)$$

\therefore Equation (i) becomes as

$$Z_1 + h_1 + \frac{V_1^2}{2g} = h_2 + \frac{V_2^2}{2g} + (i_e \times L)$$

$$\boxed{Z_1 = i_b \times L}$$

$$(i_b \times L)h_1 + \frac{V_1^2}{2g} = h_2 + \frac{V_2^2}{2g} + (i_e \times L)$$

$$i_b \times L - i_e \times L = \left(h_2 + \frac{V_2^2}{2g} \right) - \left(h_1 + \frac{V_1^2}{2g} \right)$$

$$L(i_b - i_e) = E_2 - E_1, \text{ where } E_2 = h_2 + \frac{V_2^2}{2g}$$

$$E_1 = h_1 + \frac{V_1^2}{2g}$$

$$\boxed{L = \frac{(E_2 - E_1)}{(i_b - i_e)}}$$

4. Determine the length of the back water curve caused by an afflux of 2.0 m in a rectangular channel of width 40m and depth $h_1 = 2.5$ m. The slope of the bed is given as 1 in 11000. Take Manning's $N = 0.03$

Solution:

$$b = 40 \text{ m.}$$

$$h_2 = 2 + 2.5 = 4.5 \text{ m}$$

$$\text{Manning's } N = 0.03$$

$$\boxed{\text{Afflux} = (h_2 - h_1)} = 2 \text{ m. } i_b = \frac{1}{11000} = 0.0000909$$

$$\text{Area of flow at section 1, } A_1 = b \times h_1 = 40 \times 2.5 = 100 \text{ m}^2$$

$$\text{Wetted perimeter, } P_1 = b + 2h_1 = 40 + 2.5 = 45 \text{ m}$$

$$\therefore \text{Hydraulic mean depth, } m_1 = \frac{A_1}{P_1} = \frac{100}{45} = 2.22 \text{ m}$$

$$\boxed{V = \frac{1}{N} m^{2/3} i_b^{1/2}}$$

Using Manning's formula,

$$V = \frac{1}{N} R^{2/3} S^{1/2}$$

Velocity at section1, $V = \frac{1}{N} m^{2/3} i b^{1/2}$

$$V_1 = \frac{1}{0.03} \times (2.22)^{2/3} (0.0000909)^{1/2} = \frac{1}{0.03} \times 1.7 \times 0.009534$$

$$= 0.54 \text{ m/s.}$$

Specific energy at Xⁿ1,

$$E_1 = \frac{V_1^2}{2g} + h_1$$

$$= \frac{0.54^2}{2 \times 9.81} + 2.5 = 2.5148 \text{ m.}$$

From continuity, velocity at Xⁿ is given as,

$$V_1 A_1 = V_2 \times A_2$$

$$V_2 = \frac{V_1 \times A_1}{A_2} = \frac{0.54 \times 100}{b \times h_2} = \frac{0.54 \times 100}{40 \times 4.5}$$

$$= 0.3 \text{ m/s.}$$

where area $A_2 = b \times h_2 = 40 \times 4.5 = 180 \text{ m}^2$.

Wetted Perimeter at section 2, $P_2 = b + 2h_2 = 40 + 2 \times 4.5$

$$= 49 \text{ m.}$$

$$m_2 = \frac{A_2}{P_2} = \frac{180}{49} = 3.673 \text{ m.}$$

Specific energy at section 2,

$$E_2 = h_2 + \frac{V_2^2}{2g}$$

$$E_2 = 4.5 + \frac{0.3^2}{2 \times 9.81} = 4.504 \text{ m.}$$

To find average velocity (V_{av}), find average depth (h_{av})

$$h_{av} = \frac{h_1 + h_2}{2} = \frac{2.5 + 4.5}{2} = 3.5m$$

$$V_{av} \times A_{av} = V_1 \times A_1$$

$$V_{av} = \frac{V_1 A_1}{A_{av}} = \frac{V_1 \times b \times h_1}{2} = \frac{V_1 \times h_1}{h_{av}}$$

$$= \frac{0.54 \times 2.5}{3.5} = 0.3857 \text{ m / s}$$

$$m_{av} = \frac{m_1 + m_2}{2} = \frac{2.22 + 3.673}{2} = 2.9465$$

To find the value of i_e , use Manning's formula as,

$$V_{av} = \frac{1}{N} m_{av}^{2/3} \times i_e^{1/2}$$

$$0.3857 = \frac{1}{0.03} \times 2.9465^{2/3} \times i_e^{1/2}$$

$$i_e = \left(\frac{0.3857}{68.534} \right)^2 = 0.00003167$$

The length of back water curve (L) is obtained

$$L = \frac{E_2 - E_1}{i_b - i_e} = \frac{4.504 - 2.5148}{0.0000909 - 0.00003167}$$

$$= \frac{1.9892}{0.00005923} = 33.584.3 \text{ m.}$$

5. With a neat sketch give the gradually varied flow profiles (or) briefly describe water curves or flow profiles?

Description of water curves or profiles:

A brief description of different types of flow profiles is given below. The profiles near the critical depth and channel bottom are shown by dotted lines, as at these points the streamlines are curved. And such equations of gradually varied flow are not applicable.

a. Mild slope profiles Or M profile:

A flow, in which the normal depth (h_n) is greater than the critical depth (h_c) is called streaming flow and the slope of free water surface is called mild slope or M-profile. There are three types of such profiles as discussed below.

1. M_1 -profile. It is the most important among all the profiles and represents the back water curve. This type of profile usually occurs, when a dam or weir is constructed across a mild long channel. In this case $y > y_n > y_o$ $h > h_n > h_c$
2. M_2 – profile. It represents a drawdown curve. This type of profile usually occurs, when the tail of a mild channel is submerged into a reservoir of a depth less than the normal depth. It also occurs, when the cross-section of a mild channel is subjected to a sudden enlargement. In this case $y_n > y > y_c$ $h_n > h > h_c$
3. M_3 - profile: It also represents a backwater curve. This type of profile usually occurs, when a channel after flowing below a sluice flows over a mild channel. In this case $y_n > y_c > y$.
4. **b. Steep slope profiles Or S profile:**

A flow, in which the critical depth (y_c) is greater than the normal depth (y_n) is called a rapid flow and the slope of free water surface is called steep slope or S-profile. There are three types of such profiles as discussed below.

1. S_1 - profile. It represents a back water curve. This type of profile usually occurs, when a dam or weir is constructed across steep channel. It also occurs

when the tail of a steep channel is submerged into a reservoir of a depth more than the normal depth. In this case

2. S_2 – profile. It represents a drawdown curve. This type of profile usually occurs, when the steep slope of channel changes from steep to steeper. It also occurs, when the cross-section of a steep channel is subjected to a sudden enlargement. In this case
3. S_3 - profile. It also represents the back water curve. This type of profile usually occurs, when a channel after flowing below a sluice flows over a steep channel. It also occurs when the slope of the channel changes from steeper to steep. In this case.

C. Critical slope profiles. Or C- profile.

A flow, in which the normal depth (y_n) is equal to the critical depth (y_c) is called a critical slope or C-profile. There are two types of such profiles as discussed below.

1. C_1 -profile. It represents a backwater curve. This type of profile usually occurs on the critical slope portion, when the slope of the channel changes from critical to mild. In this case, But $y_c = y_n$

2. C_2 - profile. Since in; a critical slope profile, the normal depth line and critical depth line coincide, therefore no curve is possible between these lines. However, a line coinciding with these two lines can be drawn to represent C_2 profile which will indicate a uniform critical flow. In this case $y_n = y = y_c$. Some authors do not mention the C_2 profile.

3. C_3 . Profile. It also represents a backwater curve. This type of profile, usually, occurs at the hydraulic jump. It also occurs, when the channel after flowing below a sluice gate flows over a critical slope channel. In this case $y_c > y$. But $y_c = y_n$

d. Horizontal slope profiles Or H- Profile

In a channel with horizontal bed, the normal depth (y_n) of flow is not definite and it may be either below or above the critical depth (y_c). The slope of free water surface is called horizontal slope or S- profile. There are two types of such profiles as discussed below.

1. H2-profile. It represents a drawdown curve and is similar to M_2 -profile. In this case $y_n > y > y_c$
2. H3 –profile. It represents a backwater curve and is similar to M_3 - profile. In this case $y_n > y > y_c$.

e. Adverse slope profiles (or) A - profile

In a channel with adverse slope, the bed of channel rises in the direction of flow. As a result of this, there is no definite normal depth line, and it is assumed to be above the critical depth line. The slope of free water surface is called adverse slope or A-profile. There are two types of such profiles as discussed below:

1. **A2 – profile.** It represents a drawdown curve. This type of profile usually occurs, when the cross-section of an adverse channel is subjected to sudden enlargement. In this case, $y_n > y > y_c$
2. **A3 – profile.** It represents a backwater curve. This type of profile usually occurs, when a channel after flowing below a sluice flows over an adverse slope. In this case $y_n > y_c > y$.

Channel category	Symbol	Characteristic condition	Representation
Mild slope or M slope	M	$h_n > h_c$	
Critical slope or C slope	C	$h_n = h_c$	

Steep slope or S- slope	$S_o < 0$ $h_n < h_c$ $h_n = \alpha$	
Horizontal slope or H - slope	bed slope = 0	
Adverse slope or A – Slope	<p>or -Ve</p> bed slope = - Ve	

S_o -bottom slope
 S_c -Critical slope

h_n - normal depth
 h_c - Critical depth

6. Briefly explain Graphical Integration Method.

The basic differential equation of GVF can be written.

$$\boxed{\frac{dx}{dh} = \frac{1 - \frac{V^2}{gh}}{i_b - i_e}} \quad \text{----- (1)} \quad \boxed{V = \frac{Q}{A}} \quad A = b \times h$$

$$\boxed{h = \frac{A}{b}}$$

$$\boxed{\frac{dx}{dh} = \frac{1 - \frac{Q^2 b}{gA^3}}{i_b - i_e}} \quad \text{----- (2)} \quad \frac{V^2}{gh} = \frac{Q^2 b}{A^2 gA} = \frac{Q^2 b}{gA^3}$$

For a constant Q, S_b , N and b given channel geometry the right hand side of equation (2) is a function of y.

$$\boxed{\frac{dx}{dy} = f(y)} \quad \text{----- (3)}$$

$$\text{Length of Back Water Curve} = x_2 - x_1 = \int_{y_1}^{y_2} f(y) dy$$

$$\boxed{\text{Length of BWC} = x_2 - x_1 = \int_{x_1}^{x_2} dx = \int_{y_1}^{y_2} f(y) dy}$$

If a curve is plotted between $f(y)$ & y the area under the curve between ordinates at y_1 & y_2 is the value of $\int_{y_1}^{y_2} f(y) dy$

$$V = \frac{1}{n} R^{2/3} S_f^{1/2}$$

In this method various values of y are assumed and corresponding values of $f(y)$ are calculated using

$$\frac{1 - \frac{V^2}{gy}}{S_b - S_e}$$

A curve of y Vs $f(y)$ is then plotted.

To find the distance between two section with depth. y_1 & y_2 are determined by a planimeter or by Simpson's rule.

This method can be used with equal efficiency for both natural and artificial channel.

CE2253- APPLIED HYDRAULIC ENGINEERING
(FOR IV – SEMESTER)

UNIT – IV HYDRAULIC JUMPS

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UNIT IV

HYDRAULIC JUMPS

Hydraulic jump – Types – Energy dissipation – Surges – Surge channel transitions

TWO MARKS QUESTIONS AND ANSWERS

1. Define hydraulic jump?

The rise of water level which takes place due to transformation of the unstable shooting flow (Super-critical) to the stable streaming flow (Sub-critical flow). The hydraulic jump converts kinetic energy of stream rapidly flowing into potential energy.

When hydraulic jump takes place a loss of energy due to eddy formation and turbulence occurs. Hydraulic jumps are also known as standing waves.

2. Explain hydraulic jump?

The height of water at the section 1-1 is small. Towards downstream the height or depth of water increases rapidly over a short length of the channel. This is because at the section 1-1 the flow is a shooting flow as the depth of water at section 1-1 is less than critical depth. Shooting flow is an unstable type of flow and does not continue on the downstream side. Then this shooting will convert into a streaming or tranquil flow and hence depth of water will increase. This sudden increase of depth of water is called a hydraulic jump or a standing wave.

3. What are the assumptions made for deriving an expression for the depth of hydraulic jump?

The following assumptions are made:

1. The flow is uniform and pressure distribution is due to hydrostatic before and after the jump.

2. Losses due to friction on the surface of the bed of the channel are small and hence neglected.
3. The slope of the bed of the channel is small. so that the component of the fluid in the direction of flow is negligibly small.
4. The momentum correction factor is unity.

4. Give the expression for depth of hydraulic jump.

$$d = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}}$$

$$\boxed{q = V_1 d_1} \Rightarrow \boxed{V_1 = \frac{q}{d_1}}$$

$$= -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2 \times (V_1 d_1)^2}{gd_1}}$$

d_1 \longrightarrow depth of flow at X^1-1
 d_2 \longrightarrow depth of flow at X^2-2
 V_1 \longrightarrow Velocity of flow at 1-1
 V_2 \longrightarrow Velocity of flow at 2-2
 q \longrightarrow Discharge per unit width

$$\boxed{d = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2 \times V_1^2 d_1}{g}}}$$

$$\boxed{\text{Depth of hydraulic jump} = (d_2 - d_1)}$$

5. Define length of hydraulic jump.

It is defined as the length between the two sections where one section is taken before the hydraulic jump and the second section is taken immediately after the jump. For a rectangular channel from experiments, it has been found to be equal to J to Y times the height of the hydraulic jump.

$$L_j = 5 \text{ to } 7 H_j$$

6. The depth of flow of water, at a certain section of a rectangular channel of 4m wide, is 0.5m. This discharge through the channel is 16 m³/s. If a hydraulic jump takes place on the downstream side, find the depth of flow after the jump.

Solution:

Given,

Width of channel,	$b = 4\text{m}$
Depth of flow be jump,	$d_1 = 0.5\text{ m}$
Discharge,	$Q = 16\text{m}^3 / \text{s}.$

$$\therefore \text{Discharge per unit width, } q = \frac{Q}{b} = \frac{16}{4} = 4\text{m}^2 / \text{s}$$

Let the depth of flow after jump = d_2

$$\begin{aligned} \text{Depth of flow after the jump } d_2 &= -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2g^2}{gd_1}} \\ &= -\frac{0.5}{2} + \sqrt{\frac{0.5^2}{4} + \frac{2 \times 4^2}{9.81 \times 0.5}} = -0.25 + \sqrt{0.0625 + 6.5239} \\ &= -0.25 + 2.566 = 2.316_m \end{aligned}$$

7. A sluice gate discharges water into a horizontal rectangular channel with a velocity of 10m/s and depth of flow of 1m. Determine the depth of flow after the jump and consequent loss in total head.

Given,

Velocity of flow before hydraulic jump,	$V_1 = 10\text{ m/s}$
Depth of flow before hydraulic jump,	$d_1 = 1\text{m}$
Discharge per unit width jump	$q = V_1 \times d_1 = 10 \times 1 = 10\text{m}^2 / \text{s}$

The depth of flow after jump is given by equation as

$$\begin{aligned} d_2 &= -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}} = -\frac{1.0}{2} + \sqrt{\frac{1^2}{4} + \frac{2 \times 10^2}{9.81 \times 1}} \\ &= -0.5 + \sqrt{0.25 + 20.387} = 4.043\text{m} \end{aligned}$$

Loss in total head in given by

$$h_L = \frac{(d_2 - d_1)^3}{4d_1d_2} = \frac{(4.043 - 1.0)^3}{4 \times 1.0 \times 4.043} \\ = 1.742 \text{ m.}$$

8. A hydraulic jump forms at the downstream end of spillway carrying $17.93 \text{ m}^3/\text{s}$ discharge. If the depth before jump is 0.80, determine the depth after the jump and energy loss.

Given,

Discharge $Q = 17.93 \text{ m}^3/\text{s}$.

Depth before jump, $d_1 = 0.8 \text{ m}$.

Taking width $b = 1 \text{ m}$,

Discharge per unit width, $q = \frac{17.93}{1} = 17.93$

d_2 = Depth after jump, h_L = Loss of energy

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}} = -\frac{0.8}{2} + \sqrt{\frac{0.8^2}{4} + \frac{2 \times 17.93^2}{9.81 \times 0.8}} \\ = -0.4 + \sqrt{0.16 + 81.927} = -0.4 + 9.06 = 8.66 \text{ m}$$

Loss of energy

$$h_L = \frac{(d_2 - d_1)^3}{4d_1d_2} = \frac{(8.66 - 0.8)^3}{4 \times 0.8 \times 8.66} = 17.52 \text{ m}$$

9) What is undular jump?

The Froude number F_1 ranges from 1 to 1.7 and the liquid surface shows undulations of gradually decreasing size.

10) Define Weak Jump.

F_1 ranges from 1.7 to 2.5, number of small ripples appears on the surface of the jump, and the downstream liquid surface remains smooth. The energy loss in the jump is low.

11) What is oscillating jump?

For F_1 ranging between 2.5 to 4.5, there is an oscillating jet which enters the jump bottom and oscillates to the surface. Each oscillation produces a large wave of irregular period and does extensive damage to the canal bed and banks while traveling miles downstream.

12) What is steady and strong jump?**Steady jump:**

This type of jump occurs in the Froude number range of 4.5 to 9.0. The fluctuations in the tail water depth have a very little effect on the position and the action of the jump. The energy dissipation may be in the range of 45 to 70%.

Strong jump:

For Froude number greater than 9.0, the surface down stream of the jump is rough and the energy dissipation may be upto 85%.

13) What are the elements and characteristics of a Hydraulic jump?

The following quantities are generally known as the elements of the hydraulic jump:

1. Pre-jump depth y_1
2. Post-jump depth y_2
3. Ht of the jump, $H_j = y_2 - y_1$
4. Length of the jump, $L_j = 5 H_j$
5. Specific energies before and after the jump ($E_1 - E_2$).
6. Loss of energy (ΔE) in the jump

14) What are surges?

If the flow in a channel is increased by sudden increase in the opening of sluice gate, a wave is formed which travels towards downstream. On the other hand, if the flow is decreased by sudden partial closure of the gate, the waves so formed travel upstream of the gate. Such waves, having a significant height are known as surges or surge waves.

15) What are the applications of hydraulic jump?

- It is used for dissipating excess of energy of water flowing over spillways and the hydraulic structures as through sluices and thus preventing possible erosion on the downstream of these structures.
- It raises the water level in the channels for irrigation, etc.

- It increases the weight on an apron of a hydraulic structure due to increased depth of flow and hence the lift pressure acting on the apron is considerably counter balanced.
- It increases the discharge through a slice by holding back the tail water.
- It may be used for mixing chemicals in water or other liquids since it allows through mixing due to turbulence created in it.
- To aerate water for city water supply.
- To remove air pockets from the water supply line and thus prevent air locking.

16) What is water Hammer?

When a liquid flow in a long pipe line is reduced suddenly, due to compressibility of the liquid, the sudden change in momentum would cause a pressure surge to develop. This pressure moves through phenomenon is known as water hammer and is of importance in all major pipe line designs.

17) What is positive surge and negative surge?

A surface is known as a positive surge or an elevation wave when it causes an increase in depth in direction of its travel. Positive surges are produced when a dam fails or there is sudden decrease of supply at the upstream end of channel due to lifting up regulating gate.

A negative surge is one which causes decrease in depth. It generally occurs due to sudden decrease in supply of flow upstream such as that caused by closing of head gate in a canal.

18. Give the diagram representing the propagation of surges?

19) What do you mean by propagation of surges?

Whenever there is a sudden change in the discharge or depth or both in an open channel, a rapidly varied unsteady occur during sudden operation of a control gate.

A surge producing a increase in depth is known as positive surge and the one which causes a decrease in depth is known as negative surge.

Positive surges have steep fronts, more like a hydraulic jump and the shape of the wave does not change during its translation. They are also known as moving hydraulic jumps. These are relatively easy to analyze than negative surges.

20) Give the formula for head loss in terms of Froude number and depth?

$$h_L = \frac{[d_2 - d_1]^3}{4d_1d_2}$$

$$d_2 = -\frac{d_1}{2} + \frac{d_1}{2} \sqrt{1 + 8(F_e)_1^2} = \frac{d_1}{2} \left(\sqrt{1 + 8(F_e)_1^2} - 1 \right)$$

16 Marks Questions and Answers

1 a) Derive the expression for loss of energy due to Hydraulic jump.

When hydraulic jump takes place, a loss of energy due to eddies formation and turbulence occur. This loss of energy is equal to the difference of specific energies at sections 1-1 and 2-2.

Or loss of energy due to hydraulic jump, for

$$\begin{aligned}
 h_L &= E_1 - E_2 \\
 &= \left(d_1 + \frac{V_1^2}{2g} \right) - \left(d_2 + \frac{V_2^2}{2g} \right) \\
 &= \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g} \right) - (d_2 - d_1) = \left(\frac{q^2}{2gd_1^2} - \frac{q^2}{2gd_2^2} \right) - (d_2 - d_1) \\
 h_L &= \frac{q^2}{2g} \left(\frac{1}{d_1^2} - \frac{1}{d_2^2} \right) - [d_2 - d_1] \\
 &= \frac{q^2}{2g} \left(\frac{d_2^2 - d_1^2}{d_1^2 d_2^2} \right) - [d_2 - d_1] \\
 q^2 &= gd_1 d_2 \frac{(d_2 + d_1)}{2} \tag{1}
 \end{aligned}$$

Substituting the value of q^2 in equation (1) we get,

$$\begin{aligned}
 \text{Loss of energy, } h_L &= gd_1 d_2 \frac{(d_2 + d_1)}{2} \times \frac{d_2^2 - d_1^2}{2gd_1^2 d_2^2} - (d_2 - d_1) \\
 &= \frac{(d_2 + d_1)(d_2^2 - d_1^2)}{4d_1 d_2} - (d_2 - d_1) = \left[\frac{(d_2 + d_1)^2}{4d_1 d_2} - 1 \right]
 \end{aligned}$$

$$= (d_2 - d_1) \left[\frac{d_2^2 + d_1^2 + 2d_1d_2 - 4d_1d_2}{4d_1d_2} \right] = (d_2 - d_1) = \frac{[d_2 - d_1]^3}{4d_1d_2}$$

$$h_L = \frac{[d_2 - d_1]^3}{4d_1d_2}$$

b) Derive the expression for depth of hydraulic jump in terms of .

Upstream Froude Number:

V_1 = Velocity of flow on the upstream side.

d_1 = depth of flow on upstream side.

Froude number $(F_e)_1$ on the upstream side of the jump is given by,

$$(F_e)_1 = \frac{V_1}{\sqrt{gd_1}}$$

The depth of flow after the hydraulic jump is d_2 and it is given by.

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2V_1^2 d_1}{g}} = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} \left(1 + \frac{8V_1^2}{gd_1} \right)}$$

$$= -\frac{d_1}{2} + \frac{d_1}{2} \sqrt{1 + \frac{8V_1^2}{gd_1}}$$

$$(F_e)_1 = \frac{V_1}{\sqrt{gd_1}} \text{ (or) } (F_e)_1^2 = \frac{V_1^2}{gd_1}$$

$$\text{Subs in } d_2 = -\frac{d_1}{2} + \frac{d_1}{2} \sqrt{1 + 8(F_e)_1^2} = \frac{d_1}{2} \left(\sqrt{1 + 8(F_e)_1^2} - 1 \right)$$

2) The depth of flow of water, at a certain section of a rectangular channel of 2m wide, is 0.3m. The discharge through the channel is $1.5 \text{ m}^3/\text{s}$. Determine whether a hydraulic jump will occur, if 2, find its height and loss of energy per kg of water.

Given,

$$\begin{aligned} d_1 &= 0.3 \text{ m} & Q &= 1.5 \text{ m}^3/\text{s} \\ b &= 2 \text{ m} \end{aligned}$$

Discharge per unit width, $q = \frac{Q}{b} = \frac{1.5}{2.0} = 0.75 \text{ m}^2/\text{s}$

Hydraulic jump will occur if the depth of flow on the upstream side is less than the critical depth on upstream side or if the froude number on the upstream side is more than one.

$$\text{Critical depth, } h_c = \left(\frac{q^2}{g} \right)^{1/3} = \left(\frac{0.75^2}{9.81} \right)^{1/3} = 0.3859$$

Now the depth on the upstream side is 0.3 m. This depth less than critical depth and hence hydraulic jump will occur.

$$\begin{aligned} \text{The depth of flow } d_2 &= -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}} \\ &= -\frac{0.3}{2} + \sqrt{\frac{0.3^2}{4} + \frac{2 \times 0.75^2}{9.81 \times 0.3}} = -0.15 + \sqrt{0.0255 + 0.3822} \\ &= -0.15 + 0.6362 = 0.4862 \text{ m} \end{aligned}$$

$$\therefore \text{Height of hydraulic jump} = d_2 - d_1 = 0.4862 - 0.3 = 0.1862 \text{ m}$$

Loss of energy per kg of water due to hydraulic jump is

$$\begin{aligned} h_L &= \frac{(d_2 - d_1)^2}{4d_1d_2} = \frac{[0.4862 - 0.3]^2}{4 \times 0.4862 \times 0.3} = \frac{0.1862^2}{4 \times 0.4862 \times 0.3} \\ &= 0.01106 \text{ m} - \text{kg/kg}. \end{aligned}$$

3) A sluice gate discharge water into a horizontal rectangular channel with a velocity of 6 m/s and depth of flow is 0.4m. the width of the channel is 8m. Determine whether a hydraulic jump will occur, and if so, find its height and loss of energy per kg of water. /Also determine the power lost in the hydraulic jump

Given,

$$\begin{aligned} \text{Velocity of flow, } V_1 &= 6 \text{ m/s.} & b &= 8 \text{ m} \\ d_1 &= 0.4 \text{ m.} \end{aligned}$$

$$\begin{aligned} q &= \frac{Q}{b} = \frac{V_1 \times \text{area}}{b} = \frac{V_1 \times d_1 \times b}{b} = V_1 \times d_1 = 6 \times 0.4 \\ &= 2.4 \text{ m}^2 / \text{s} \end{aligned}$$

Froude number on the upstream side,

$$(F_e)_1 = \frac{V_1}{\sqrt{gd_1}} = \frac{6.0}{\sqrt{9.81 \times 0.4}} = 3.0289 = 3.029$$

As froude number is more than one, the flow is shooting on the upstream side. Shooting flow is unstable flow and it will convert itself into streaming flow by raising its height and hence hydraulic jump will take place.

ii. Let the depth of hydraulic jump = d_2

$$\begin{aligned} d_2 &= \frac{d_1}{2} + \sqrt{1 + 8(F_e)_1^2 - 1} = \frac{0.4}{2} \left(\sqrt{1 + 8 \times 3.029^2} - 1 \right) \\ &= 1.525 \text{ m.} \end{aligned}$$

$$\therefore \text{Ht of hydraulic jump } d_2 - d_1 = 1.525 - 0.4 = 1.125 \text{ m}$$

iii. Loss of energy per kg of water

$$h_L = \frac{(d_2 - d_1)^3}{4d_1d_2} = \frac{[1.525 - 0.4]^3}{4 \times 0.4 \times 1.525} = 0.5835 \text{ m} - \text{kg / kg of water}$$

iv Power lost in $KW = \frac{pg \times Q \times h_L}{1000}$, where $Q = V \times \text{area}$

$$\begin{aligned} V_1 \times d_1 \times b &= 6 \times 0.4 \times 8 = 19.2 \text{ m}^3 / \text{s} \\ \therefore \text{Power, } P &= \frac{1000 \times 9.81 \times 19.2 \times 0.5835}{1000} = 109.9 \text{ Kw.} \end{aligned}$$

4) What are the elements and characteristics of a Hydraulic jump?

The following quantities are generally known as the elements of the hydraulic jump:

7. Pre-jump depth y_1
8. Post-jump depth y_2
9. Ht of the jump, $H_j = y_2 - y_1$
10. Length of the jump, $L_j = 5 H_j$
11. Specific energies before and after the jump ($E_1 - E_2$).
12. Loss of energy (ΔE) in the jump

The following dimensionless quantities are generally known as the characteristics of the jump:

1. Relative loss ($\Delta E/E_1$): It is defined as the ratio of energy loss and the specified energy before the jump.

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

$$\frac{\Delta E}{y_1} = \frac{\left(\frac{y_2}{y_1} - y_1\right)^3}{4 \frac{y_2}{y_1}} \quad (1)$$

$$E_1 = y_1 + \frac{V_1^2}{2g}$$

$$\frac{E_1}{y_1} = 1 + \frac{V_1^2}{2gy_1} = 1 + \frac{F_1^2}{2}, \quad (2)$$

$$\frac{y_2}{y_1} = \frac{1}{2} \left[\sqrt{1 + 8F_1^2} - 1 \right] \quad (3)$$

from 1,2-3 equations $\frac{\Delta E}{E_1} = \frac{\left(\sqrt{1 + 8F_1^2} - 3\right)^3}{8\left(F_1^2 + 2\right)\left(\sqrt{1 + 8F_1^2} - 1\right)}$

The relative loss being a ratio of energies is dimensionless.

2. Efficiency of the jump $\left(\frac{E_2}{E_1}\right)$: The ratio of the specific energies after and before the jump is known as the efficiency of the jump. The efficiency may be expressed in terms Froude number F_1 , thus

$$\frac{E_2}{E_1} = \frac{1 + (8F_1^2 + 1)^{3/2} - 4F_1^2}{8F_1^2(2 + 8F_1^2)}$$

3. Relative Height of the Jump (H_j/E_1); The height of the jump is defined as difference between the depths after and before the jump

$$H_j = y_2 - y_1$$

Since E_1 is the specific energy per unit weight of liquid, it has the dimensions of length and hence jump height may be made dimensionless by dividing it by E_1

$$\frac{H_j}{E_1} = \frac{y_2 - y_1}{E_1} = \frac{\frac{y_2}{y_1} - 1}{E_1 / y_1} \Rightarrow$$

$$\boxed{\frac{H_j}{E_1} = \frac{\sqrt{1 + 8F_1^2} - 3}{2 + F_1^2}}$$

5. What are the various types of hydraulic jumps?

For jumps in which the y_2 / y_1 is not greater than 2.0, the liquid surface does not rise abruptly and has a number of undulations of gradually diminishing size. Such jump is called a sub-undular jump.

For $y_2 / y_1 = 2$, the upstream Froude number $F_1 = \sqrt{3}$. This fixes the upper limit of F_1 for undular jumps.

For higher values of y_2 / y_1 ($y_2 / y_1 > 2$ and consequently $F_1 > \sqrt{3}$) the liquid surface rises fairly abruptly and the hydraulic jump then is known as a direct jump.

The United States Bureau of Reclamation has classified the jump into the following five categories, depending upon the magnitude of the Froude number of the approaching flow F_1 surface undulations.

1. **UNDULAR JUMP:** The Froude number F_1 ranges from 1 to 1.7 and the liquid surface shows undulations of gradually decreasing size.

2. **WEAK JUMP:** F_1 ranges from 1.7 to 2.5, number of small ripples appear on the surface of the jump, and the downstream liquid surface remains smooth. The energy loss in the jump is low.

3. OSCILLATING JUMP: For F_1 ranging between 2.5 to 4.5, there is an oscillating jet which enters the jump bottom and oscillates to the surface. Each oscillation produces a large wave of irregular period and does extensive damage to the canal bed and banks while traveling miles downstream.

4. STEADY JUMP: This type of jump occurs in the Froude number range of 4.5 to 9.0. The fluctuations in the tail water depth have a very little effect on the position and the action of the jump. The energy dissipation may be in the range of 45 to 70%.

5. STRONG JUMP: For Froude number greater than 9.0, the surface down stream of the jump is rough and the energy dissipation may be upto 85%.

6) Briefly explain positive surge moving downstream and upstream?

Positive surge moving downstream

simulated steady flow.

Absolute velocity	V_1	\longrightarrow	Depth after surge (jump)	\longrightarrow	y_2
Approach velocity	$(V_w - V_1)$	\longrightarrow	velocity after surge (jump)	\longrightarrow	$(V_w - V_2)$
Approach depth		\longrightarrow	y_1		

The continuity equation is,

$$A_1 (V_w - V_1) = A_2 (V_w - V_2)$$

By considering unit width of the rectangular channel,.

$$y_1 (V_w - V_1) = y_2 (V_w - V_2)$$

By application of momentum equation

$$\frac{(V_w - V_1)^2}{gy_1} = \frac{1}{2} \frac{y_2}{y_1} \left(\frac{y_2}{y_1} + 1 \right)$$

POSITIVE SURGE MOVING UPSTREAM.

This kind of surge occurs on the upstream of a slice gate when the gate is closed suddenly and in the phenomenon of tidal tores. (In a tidal river, a rising tide in the sea causes a surge to form, known as tidal bore, which travels upstream raising the depth of water in the river).

The continuity equation is $A_1(V_w - V_1) = A_2(V_w - V_2)$

For unit width of the rectangular channel

$$y_1(V_w + V_1) = y_2(V_w + V_2)$$

Initial velocity $\longrightarrow (V_w + V_1)$

Initial depth $\longrightarrow y_1$

Final velocity $\longrightarrow (V_w + V_2)$

Depth after surge $\longrightarrow y_2$

$$\frac{(V_w + V_1)^2}{gy_1} = \frac{1}{2} \left(\frac{y_2}{y_1} \right) \left(1 + \frac{y_2}{y_1} \right)$$

Other form of equation:

$$\frac{(V_w - V_2)^2}{gy_2} = \frac{1}{2} \frac{y_1}{y_2} \left(1 + \frac{y_1}{y_2} \right)$$

(or)

$$\frac{(V_w + V_2)^2}{gy_2} = \frac{1}{2} \frac{y_1}{y_2} \left(1 + \frac{y_1}{y_2} \right)$$

7 a) A 2.0 m wide rectangular channel has a flow with a velocity of 2m/s and depth of 1.3m. The rate of inflow at the upstream and is suddenly increased of an extent that the depth is doubled in magnitude. Estimate the absolute velocity of the resetting surge and the new discharge.

Solution:

Conditions of relative steady flow are simulated by adding the velocity $-V_w$ vertically (ie) to the left in fig.

Surge moving downstream.

$$y_1 = 1.30\text{m}, \quad y_2 = 2.60\text{m}, \quad V_1 = 2\text{m/s}$$

$$\frac{V_w - V_1}{\sqrt{gy_1}} = \frac{1}{2} \left[\frac{y_2}{y_1} \left(\frac{y_2}{y_1} + 1 \right) \right]^{1/2}$$

$$V_w - 2 = \sqrt{9.81 \times 1.3} \left[\frac{1}{2} \times 2 \times (2 + 1) \right]^{1/2} = 6.185$$

$$V_w - 2 = 8.185 \text{ m/s. (in simulated flow).}$$

The surge moves downstream with a velocity of 8.185 m/s.

$$\text{By continuity equation:} \quad y_1(V_w - V_1) = y_2(V_w - V_2)$$

$$1.3(8.185 - 2) = 2.6(8.185 - V_2)$$

$$V_2 = 8.185 - 3.0925 = 5.093 \text{ m/s}$$

$$\begin{aligned} \text{New discharge } Q_2 &= By_2V_2 \\ &= 2 \times 2.6 \times 5.093 \\ &= 26.484 \text{ m}^3/\text{s} \end{aligned}$$

7. B) A rectangular channel is discharging $1.60 \text{ m}^3/\text{s}$ per metre width at a depth of 0.8 m . If a sudden release of flow at the upstream end double the discharge intensity, determine the absolute velocity of the resulting surge and the new depth.

Solution:

$$Y_1 = 0.8 \text{ m} \quad V_1 = \frac{1.6}{0.8} = 2.0 \text{ m/s}$$

$$V_2 y_2 = 2 \times 1.6 = 3.2 \Rightarrow V_2 = \frac{3.2}{y_2}$$

By continuity equation: $y_1(V_\omega - V_1) = y_2(V_\omega - V_2)$

$$0.8(V_\omega - 2.0) = V_\omega y_2 - 3.2$$

$$V_\omega - (y_2 - 0.8) = 1.6$$

$$V_\omega = 1.6 / (y_2 - 0.8)$$

Surge equation,

$$\begin{aligned} &= \frac{(V_\omega - V_1)^2}{gy_1} = \frac{1}{2} \frac{y_2}{y_1} \left(1 + \frac{y_2}{y_1} \right) = \frac{\left(\frac{1.6}{y_2 - 0.8} - 2.0 \right)^2}{9.81 - 0.8} \\ &= \frac{1}{2} \frac{y_2}{0.8} \left(1 + \frac{y_2}{0.8} \right) = \left(\frac{3.2 - 2y_2}{y_2 - 0.8} \right)^2 = 6.131 y_2 (0.8 + y_2) \end{aligned}$$

By trial & Error, $y_2 = 1.088 \text{ m}$.

$$V_2 = \frac{3.2}{y_2} = \frac{3.2}{1.088} = 2.941 \text{ m/s}$$

$$V_\omega = \frac{1.6}{(1.088 - 0.8)} = 5.556 \text{ m/s}$$

$V_\omega = +5.556 \text{ m/s}$ in simulated flow.

Hence the surge moves downstream with a velocity of 5.556 m/s .

8. Give a brief note on surge tanks.

Whenever a valve, fitted at the end of a pipe is suddenly closed, it causes hammer flow in the pipeline. Moreover, in hydro-electric power plants, since the requirement of water goes on changing it is therefore, essential to increase or decrease the discharge flowing through the pipe line.

A little consideration will show that whenever the requirement of water is suddenly decreased, the valve is suddenly closed as a result of which the entire pipe length between the reservoir and the turbine will experience an increased pressure.

In order to overcome the above mentioned problems, a storage reservoir is fitted at some opening made on the pipeline (Called pen-stock) in order to store water when the valve is suddenly closed, or to discharge water when increased discharge is required. Such a storage reservoir is called surge tank, which has the following two functions:

1. To control the pressure variations, due to rapid changes in the pipeline flow, thus eliminating water hammer possibilities.
2. To regulate the flow of water to the turbines by providing necessary retarding head of water.

The surge tanks are placed as close to the turbine as possible, the height of surge tank is generally kept above the maximum water level in the supply level reservoir. Though there are many types of surge tanks, yet the following are important from the sulycat point of view.

1. Simple surge tank,
2. Restricted orifice type surge tank.
3. Differential surge tank.

9. A tidal estuary is flowing at the rate of 6.5 Km/hr and a depth of 2m. Owing to the tide in these, the level rapidly rase and the resulting surge or ‘bore’ took one hour to reach a spot 22.5 m up the stream. Compute the height of the bore above the initial depth of flow. What speed and direction will the flow have after the bore has passed?

$$V_1 = \frac{6.5 \times 1000}{60 \times 60} = 1.805 \text{ m/s}$$

$$y_1 = 2 \text{ m}$$

$$V\omega = \frac{22.5 \times 1000}{60 \times 60} = 6.25 \text{ m/s}$$

$$V_1 + C = \sqrt{\frac{gy^2}{2y_1}}(y_1 + y_2)$$

Replacing C by v_0

$$V_1 + V\omega = \sqrt{\frac{gy^2}{2y_1}}(y_1 + y_2)$$

$$1.805 + 6.25 = \sqrt{\frac{9.81y_2(2 + y_2)}{2 \times 2}}$$

$$(8.055)^2 = \frac{9.81y_2(2 + y_2)}{2 \times 2} \Rightarrow \frac{54.8 \times 4}{9.81} = y_2(2 + y_2)$$

$$y_2^2 + 2y_2 - 26.47 = 0$$

$$y_2 = -\frac{2 \pm \sqrt{4 + 105.88}}{2} = 4.24m$$

$$\text{Let of the bore} = y_2 - y_1 = 4.24 - 2.0 = 2.24m$$

$$\begin{aligned} \text{Flow taken in by the bore} &= (A_2 - A_1)V_w \\ &= 2.24 \times 6.25m^3 / s / m \text{ width} \\ &= 14.4m^3 / s / m \text{ width.} \end{aligned}$$

$$\text{Normal flow in the river} = 2.0 \times 1.805 = 3.61m^3 / s / m \text{ width}$$

$$\text{Net flow in the direction of bore} = 14.0 - 3.61 = 10.391m^3 / s / m$$

$$\text{Speed of flow after the bore has passed} = \frac{10.39}{y^2} = \frac{10.39}{4.24} = 2.475m / s$$

The direction of flow is opposite to the normal direction of flow.

10) A horizontal rectangular channel of 3m width and 2m water depth conveys water at 18 m³/s. If the flow rate is suddenly reduced to 2/3 of its original value, compute the magnitude and speed of the upstream ward surge. Assume that the front of the surge is rectangular and friction in the channel is neglected.

Solution:

When the discharge in the channel is suddenly reduced by operating a gate, an upstream surge will be developed which will move upstream with a celerity C as shown. A observer standing on the canal bank will notice the surge moving upstream. This unsteady flow case can be transformed into a steady one by super imposing flow with velocity C in the opposite direction.

The continuity equation may be written as,

$$by_1(V_1 + C) = by_2(V_2 + C) \quad (\text{or}) \quad y_1(V_1 + C) = y_2(V_2 + C)$$

Momentum equation,

$$\frac{by_1}{2}(y_2^2 - y_1^2) = \frac{by_1}{g} V_1 (V_1 + C)(V_1 - V_2) \quad (\text{or})$$

$$(y_2^2 - y_1^2) = \frac{2y_1}{g} (V_1 + C)(V_1 - V_2)$$

From the data given,

$$y_1 = 2\text{m}, \quad Q_1 = 18 \text{ m}^3 / \text{s}.$$

$$Q_2 = \frac{2}{3} Q_1 = \frac{2 \times 18}{3} = 12 \text{ m}^3 / \text{s}$$

$$V_1 = \frac{Q_1}{by_1} = \frac{18}{3 \times 2} = 3 \text{ m} / \text{s}$$

$$Q_2 = by_2 V_2$$

$$\therefore V_2 y_2 = \frac{12}{3} = 4 \text{ m}^3 / \text{s} / \text{m}$$

from the continuity equation,

$$V_1 y_1 = V_2 y_2 + C(y_2 - y_1)$$

$$3 \times 2 = 4C(y_2 - y_1) \quad (\text{or}) \quad C = \frac{2}{y_2 - 2}$$

Substituting in the momentum equation,

$$(y_2^2 - 4) = \frac{2 \times 2}{9.81} (3 + C)(3 - V_2)$$

$$y_2^2 - 4 = 0.41 \left(3 + \frac{2}{y_2 - 2} \right) \left(3 - \frac{4}{y_2} \right)$$

$$\text{Solving by trials, } y_2 = 2.8\text{m, \& } V_2 = \frac{4}{2.8} = 1.4286 \text{ m} / \text{s}.$$

$$\text{Heat of surge} = y_2 - y_1 = 2.8 - 2.0 = 0.8\text{m}$$

$$\text{Celerity of surge, } C = \frac{2}{2.8 - 2} = 2.5 \text{ m} / \text{s}.$$

CE2253- APPLIED HYDRAULIC ENGINEERING
(FOR IV – SEMESTER)

UNIT – V PUMPS AND TURBINES

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UNIT V

PUMPS AND TURBINES

Application of momentum principle – Impact of jets on plane and curved plates - turbines - classification - radial flow turbines - axial flow turbines – Impulse and Reaction turbines - draft tube and cavitations - performance of turbines - similarity laws - centrifugal pump - minimum speed to start the pump – multistage Pumps – Jet and submersible pumps - Positive displacement pumps - reciprocating pump - negative slip - flow separation conditions - air vessels -indicator diagram and its variation - savings in work done - rotary pumps.

TWO MARKS QUESTIONS AND ANSWERS

1) Define momentum principle (or) state momentum principle.

It states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction. The force acting on a fluid mass 'm' is given by Newton's second law of motion.

$$F = m \times a$$

Acceleration acting in the same direction as force F

2) Give the Impulse – momentum equation.

$$a = \frac{dv}{dt}$$

$$F = m \frac{dv}{dt} = d \left(\frac{mv}{dt} \right)$$

$$F = d \left(\frac{mv}{dt} \right)$$

This equation is known as the momentum principle.

$$F \cdot dt = d(mv)$$

This is known as the impulse momentum equation and states that the impulse of a force F acting on a fluid of mass m in a short interval of time dt is equal to the change of momentum d(mv) in the direction of force.

3) What is impact of jet?

Impact of jet means the force exerted by the jet on a plate which may be stationary or moving.

4) What are the cases to be considered in the impact of jet?

The following cases of the impact of jet ie, the force exerted by the jet on a plate, will be considered,

A) Force exerted by the jet on a stationary plate when

- i) Plate is vertical to the jet
- ii) Plate is inclined to the jet
- iii) Plate is curved.

B) Force exerted by the jet on a moving plate,

- i) Plate is vertical to the jet
- ii) Plate is inclined to the jet
- iii) Plate is curved.

5) Define hydraulic machines.

Hydraulic machines are defined as those machines which convert either hydraulic energy (energy possessed by water) into mechanical energy (which is further converted into electrical energy) or mechanical energy into hydraulic energy.

6) What are turbines and pumps?

Turbine:

The hydraulic machines which convert the hydraulic energy into mechanical energy are called turbines. Ex: pelton, Francis, Kaplan turbine.

Pumps:

A hydraulic machine which convert the mechanical energy into hydraulic energy is called pumps. Ex: Centrifugal pump, reciprocating pump.

7) Give the working of turbines.

Turbines are defined as the hydraulic machines which convert hydraulic energy into mechanical energy. This mechanical energy is used in running an electric generator which is directly coupled to the shaft of the turbine. Thus the mechanical energy is converted into electrical energy. The electric power which is obtained from the hydraulic energy is known as hydro electric power. At present the generation of hydro electric power is the cheapest as compared by the power generated by other sources such as oil, coals etc.

8) What are the general layouts of a hydroelectric power plant?

It consists of:

- 1) A dam constructed across a river to store water.
- 2) Pipe of large diameter called penstocks, which carry water under pressure from the storage reservoir to the turbines. These pipes are made of steel or reinforced concrete.
- 3) Turbine having different types of vanes fitted to the wheels
- 4) Tail race, which is a channel which carries water away from the turbines after the water has worked on the turbines. The surface of water in the tail race is also known as tailrace.

9) Define Gross head and net head.

Gross head:

The difference between the head race level and tailrace level when no water is flowing is known as Gross head. It is denoted by 'H_g'.

Net head:

It is also called effective head and is defined as the head available at the limit of the turbine. When water is flowing from head race to the turbine, a loss of head due to friction between the water and penstocks occurs.

$$H = H_g - h_f$$

$$H_f = \frac{4xfLxV^2}{Dx2g}$$

H_g = Gross head

V = Velocity of flow in penstock

L = Length of penstock

D = Diameter of penstock.

10) Give a neat sketch of layout of a HYDRO ELECTRIC POWER PLANT.

11) What is impulse and reaction turbine?

Impulse Turbine:

If at the inlet of turbine the energy available is only kinetic energy, the turbine is known as Impulse turbine.

Reaction turbine:

If at the inlet of turbine, water possesses kinetic energy as well as pressure energy the turbine is known as Reaction turbine.

12) What are the different flows types of a turbine?

- **Tangential flow turbine**
- **Radial flow turbine**
- **Inward radial flow turbine**
- **Outward radial flow turbine**
- **Axial flow**
- **Mixed flow**

13) What do you know about pelton wheel and its parts?

Pelton wheel is a tangential flow impulse turbine.

- 1) Water strikes the bucket along the tangent of the runner.
- 2) Kinetic energy
- 3) Turbine used for high heads.

Parts:

- 1) Nozzle & flow regulating arrangement
- 2) Runner & bucket
- 3) casing
- 4) Breaking jet

14) List out the important efficiencies of a turbine.

- Hydraulic efficiency
- Mechanical efficiency
- Volumetric efficiency
- Over all efficiency

15) Give the formula for Hydraulic and mechanical efficiency?

$$\text{Hydraulic efficiency } \eta_h = \frac{RP}{WP}$$

RP – runner power

WP- water power

SP- Shaft power.

Mechanical efficiency η_m

$$\eta_m = \frac{SP}{RP}$$

16) What is overall efficiency and volumetric efficiency?

Overall efficiency:

$$\eta_0 = \frac{SP}{WP}$$

$$\eta_0 = \eta_n \times \eta_m$$

Volumetric efficiency:

$$\eta_v = \text{Volume of water at runner} / \text{Volume of water at inlet}$$

17) What is centrifugal pump and reciprocating pump?

If the mechanical energy is converted into hydraulic energy by means of centrifugal force acting on liquid the pump is known as centrifugal pump.

If the mechanical energy is converted into hydraulic energy by sucking the liquid into a cylinder in which a Piston is reciprocating (Moving backwards and forwards) which exerts the thrust on liquid and increases its hydraulic energy is known as reciprocating pump.

18) What is slip and negative slip of a reciprocating pump?

The slip of a pump is defined as the difference between the theoretical and actual discharge of pump. The actual discharge of the pump is less than the theoretical discharge due to leakage.

If the actual discharge is more than the theoretical discharge the slip of the pump will become negative. In that case the slip of pump is known as negative slip. This occurs delivery pipe is short, suction pipe is long and pump is running at high speed.

19) What is an indicator diagram and what is the use of air vessels?

The indicator diagram is a graph between the pressure head in the cylinder and the distance traveled by the piston from inner dead center for one complete revolution of the crank.

Air vessels are used to obtain a continuous supply of water at uniform rate, to save a considerable amount of work and to run the pump at high speed without separation.

20) Sketch the main parts of a reciprocating pump.

The main parts are:

- Cylinder with a piston, piston rod, connecting rod and a crank
- Suction pipe
- Delivery pipe
- Suction valve
- Delivery valve

16 Marks Questions and Answers

1) What are the design aspects of pelton wheel turbine?

The design aspects are:

1. Velocity of jet at inlet

$$V_1 = C_v \sqrt{2gh}$$

C_v = coefficient of velocity = 0.98 or 0.99

H – Net head of turbine.

2. Velocity of wheel or peripheral velocity

$$u = \frac{\pi DN}{60} \text{ or } u = \sqrt{2gh}$$

ϕ Speed ratio = 0.43 to 0.48

3. Angle of deflection of jet is taken as 165° if not given

4. Mean diameter of wheel D can be found from

$$u = \frac{\pi DN}{60} \text{ or } D = \frac{60u}{\pi n}$$

5. Jet ratio (m): it is defined as the ratio of the pitch diameter (D) of the pelton wheel to the diameter of jet (d). It is denoted by m

$$m = \frac{D}{d}$$

6. Number of buckets or vanes on a runner:

$$Z = 15 + 0.5m$$

M – Jet ratio

7. Number of jets:

$$n = \frac{\text{Total rate of flow through the turbine}}{\text{rate of flow of water through a single jet}}$$

2. Define Radial flow reaction turbine?

Radial flow turbines are those functions in which the water flows in the radial direction. Reaction turbine means that the water at inlet of turbine possesses kinetic as well as pressure energy.

- Outward radial flow turbine ---- water flows from outwards to inwards
- Reaction turbine ----- Water at inlet possesses kinetic energy as well as pressure energy.

Casing and runner is always full of water.

Main parts of Radial flow reaction turbine:

1. Casing
2. Guide mechanism.
3. Runner
4. Draft tube (tube of increasing area used for discharge water from exit of turbine to the tail race.)

Inward Radial flow turbine:

Outer dia of runner is inlet and inner dia is the outlet.

Work done per second on the runner by water = $\rho Q = [V_{w_1} V_1 \pm V_{w_2} V_2]$

$$U_1 = \frac{\pi D_1 N}{60} \quad U_2 = \frac{\pi D_2 N}{60} \quad \text{----- (2)}$$

U_1 = tangential velocity of wheel at inlet

U_2 = tangential velocity of wheel at outlet.

D_1 = Outer diameter of runner

D_2 = Inner diameter of runner.

N = speed of turbine in rpm.

Work done per second per unit weight of water /sec = $\frac{1}{g} [V_{w_1} V_1 \pm V_{w_2} V_2]$ ----- (3)

If $\beta = 90^\circ$ then $V_{w_2} = 0$ then (3) becomes

$$= \frac{1}{g} V_{w_1} U_1$$

$$\eta_h = \frac{R.P}{W.P} = \frac{V_{w_1} V_1 \pm V_{w_2} V_2}{gH}$$

If discharge is radial at outlet, then $V_{w_2} = 0$

$$\eta_h = \frac{V_{w_1} V_1}{gH}$$

Degree of reaction:

Degree of reaction is defined as the ratio of pressure energy change inside the runner to the total energy change inside the runner.

$$R = \frac{\text{Change of pressure energy inside the runner}}{\text{Change of total energy inside the runner}}$$

This is also known as change of total energy per unit weight inside the runner.

$$H_e = 1/g [V_{w_1} U_1 \pm V_{w_2} U_2]$$

$$1. \text{ Speed ratio } K_v = \frac{V_1}{\sqrt{2gH}}$$

$$2. \text{ Flow ratio } K_f = \frac{V_{f_1}}{\sqrt{2gH}}$$

$$3. \text{ Discharge of the turbine } Q = \pi D_1 B_1 x V_{f_1} \\ = \pi D_2 B_2 x V_{f_2}$$

D_1 = diameter of runner at inlet.
 B_1 = Width of runner at inlet.
 V_{f1} = velocity of flow at inlet.
 D_2, B_2, V_{f2} = outlet.

4) If thicknesses of vanes are considered:

Area = $(\pi D_1 - n t) B_1$
 N = Number of vanes or runner.
 t = thickness of each vane.

$$Q = (\pi D_1 - n t) B_1 \times V_{f1}$$

5) Head on the turbine

$$H = \frac{P_1}{\rho g} + \frac{V_1^2}{2g}$$

6) Radial discharge:

This means the angle made by absolute velocity with the tangent on the wheel is 90° and the component of the wheel velocity is zero.

Radial discharge at outlet = $\beta = 90^\circ, V_{w2} = 0$

7) If there is no loss of energy when water flow through vanes, then,

$$\frac{H - V_2^2}{2g} = \frac{1}{g} [V_{w1} \pm V_{w2} V_2]$$

3) Briefly explain axial flow reaction turbine:

If the water flows parallel to the axis of the rotation of the shaft, the turbine is known as axial flow turbine. And if the head at inlet of the turbine is the sum of pressure energy and kinetic energy and during the flow of water through runner a part of pressure energy is converted into kinetic energy, the turbine is known as reaction turbine.

For the axial flow reaction turbine, the shaft of the turbine is vertical, the lower end of the shaft is made larger which is known as “hub” or “boss”.

The following are the important type of axial flow reaction turbines:

1. Propeller turbine.
2. Kaplan turbine.

The main parts of Kaplan turbine are:

1. Scroll casing
2. Guide vanes mechanism
3. Hub with vanes or runner of the turbine.
4. Draft tube.

$$Q = \frac{\pi}{4} (D_0^2 - D_b^2) \times V f_1$$

D_0 = outer diameter of the runner

D_b = diameter of hub

$V f_1$ = Velocity of flow inlet.

2. Peripheral velocity at inlet and outlet are equal:

$$V_1 = V_2 = \frac{\pi D_0 N}{60}, D_0 = \text{outer diameter of runner.}$$

3. Velocity of flow at inlet and outlet are equal.

$$V_{f1} = V_{f2}$$

4. Area of flow at inlet = Area of flow at outlet

$$5. \text{ Specific speed } N_s = \frac{N \sqrt{P}}{H^{\frac{5}{4}}} = \frac{\pi}{4} (D_0^2 - D_b^2)$$

4) What are the formulas used for Francis turbine?

Inward flow reaction turbine having radial discharge at outlet is known as Francis turbine.

In modern Francis turbine, the water enters the runner of the turbine in the radial direction at outlet and leaves in the axial direction at the inlet of the runner. Thus the modern Francis turbine is a mixed flow type turbine.

Inward flow Reaction turbine velocity triangles = Francis turbine

But, $V_{w2} = 0$

- 1) Work done by water on the runner/sec = $\rho Q (V_{w1} V_1)$
- 2) Work done /S/ unit weight of water striking/s = $\frac{1}{g} (V_{w1} V_1)$
- 3) Hydraulic efficiency = $\eta_h = \frac{V_{w1} U_1}{gH}$
- 4) Flow ratio = $\frac{V_{f1}}{\sqrt{2gH}} \approx 0.15 \text{ to } 0.3$
- 5) Speed ratio = $\frac{u_1}{\sqrt{2gH}} = 0.6 \text{ to } 0.9$
- 6) Ratio of width of wheel to its diameter $n = \frac{B_1}{D_1} \approx 0.1 \text{ to } 0.4$

Outward radial flow reaction turbine:

The inner diameter of the runner is inlet
Outer diameter of the runner is outlet.

Here,

$$U_1 < U_2$$

$$D_1 < D_2$$

5) What is draft tube given its purpose and types?

The pressure at the exit of runner of the reaction turbine is less than atmospheric pressure.

Thus the water at the exit of the runner cannot be directly discharged to the tail race channel.

The draft tube is a pipe of gradually increasing area which connects the outlet of the runner to the tail race. It is used for discharging water from exit of the turbine to the tail race. This pipe of gradually increasing area is called a draft tube.

One end of the draft tube is connected to the outlet of the runner while the other end is submerged below the level of water in the tail race.

Purpose of draft tube:

1. Serves as a passage for water discharge.
2. It permits a negative head to be established at outlet of the runner and thereby increase the net head on the turbine. The turbine may be placed above the tail race without any loss of net head and hence turbine may be inspected properly.
3. It converts a large proportion of kinetic energy ($V^2/2g$) rejected at the outlet of turbine into useful pressure energy. Without the draft tube, the kinetic energy rejected at the outlet of the turbine will go waste to the tail race.

Hence by using draft tube, the net head on the turbine, increases the turbine develops more power and also the efficiency of the turbine increases.

Type of draft tube:

- Conical
- Simple elbow
- Moody spreading tubes
- Elbow draft tubes with circular inlet and rectangular outlet.

Most efficient

- Conical draft tube
- Moody spreading

Require less space

- Simple elbow
- Elbow draft tubes with circular inlet and rectangular outlet.

$$\frac{p_1}{\rho g} = \frac{Pa}{\rho g} - H_s - \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_f \right)$$

Efficiency of draft tube:

$$\eta_d =$$

V_1 = velocity of water at inlet of draft tube.

V_2 = Velocity of water at outlet of Draft tub.

H_f = Head loss in the draft tube.

6) Give the significance and definition for specific speed and explain unit quantities or performance of turbine.

It is defined as the speed of a turbine which is identical in shape, geometrical dimensions blade angles, gate opening etc, with the actual turbine but of such a size that it will develop unit power when working under unit head. It is denoted by symbol N_s . The specific speed is used in comparing the different types of turbine as every type of turbine has different specific speed.

$$N_s = \frac{N\sqrt{P}}{H^{\frac{5}{4}}}$$

Where,

N = Speed of actual turbine.

P = power developed

H = Head under which the turbine is working.

Significance of specific speed:

Specific speed plays an important role for selecting the type of the turbine. Performance of a turbine can be predicted by knowing the specific speed of the turbine.

s.no	Specific speed in Rpm	Type of turbine
1	8.5 to 30	Pelton wheel with single jet
2	30 to 51	Pelton wheel with two jet
3	51 to 255	Francis turbine
4	255 to 860	Kaplan or propeller turbine

Performance of turbine or unit quantities:

In order to predict the behaviors of a turbine working under varying conditions of head, speed, output and gate opening the result are expressed in terms of quantities which may be attained when the head on the turbine is reduced to unity.

1. Unit speed
2. Unit Discharge
3. Unit power.

Unit speed:

Speed of a turbine working under a unit head.

$$N_u = \frac{N}{\sqrt{H}}$$

Unit discharge:

Discharge passing through a turbine which is working under a unit head.

$$Q_u = \frac{Q}{\sqrt{H}}$$

Unit power:

Power developed by a turbine which is working under unit head.

$$P_u = \frac{P}{H^{\frac{3}{2}}}$$

Use of unit quantities:

If a turbine is working under different head, the behaviour of a turbine can be easily known from the values of unit quantity.

H_1, H_2 ----- heads under which a turbine works.

N_1, N_2 ----- Corresponding speeds

Q_1, Q_2 ----- discharge.

P_1, P_2 ----- Power developed by the turbine.

$$N_u = \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$$Q_u = \frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}$$

$$P_u = \frac{P_1}{H_1^{\frac{3}{2}}} = \frac{P_2}{H_2^{\frac{3}{2}}}$$

7) Sketch the characteristics curves of hydraulic turbines.

With the help of characteristics curves the exact behavior and performance of the turbine under different condition can be known. These curves are plotted from the results of the test, performed on the turbine under different working condition. The important parameters which are varied during a test on turbine are:

1. Speed (N)
2. Head (H)
3. Discharge (Q)
4. Power (P)
5. Overall efficiency (η)
6. Gate opening

(H) Main characteristic curves (or) Constant Head curve

(N) Operating characteristic curve (or) Constant speed curve

(η) Muschel curves (or) Constant efficiency curve.

Operating characteristic curves (or) Constant speed curves:

These curves are plotted when the speed on the turbine is constant. It is defined as the speed of the turbine which is identical in slope geometrical dimension blade angle; gate opening etc is the actual turbine such a size that it will develop unit power when working under head. It is denoted by symbol N_s .

$$N_s = \frac{N\sqrt{P}}{H^{\frac{5}{4}}}$$

N – Speed of the actual turbine

P --- Power developed

H --- Head under which the turbine is working.

8) What is cavitation given its effects and precautions?

Cavitation:

The phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below its vapour pressure and sudden collapsing of vapour bubbles in a region of high pressure.

Precaution against cavitation:

The pressure of the flowing liquid in any part of his hydraulic system should not be allowed to fall below its vapour pressure. If the following liquid is water then the absolute pressure head should not be below 2.5m of water.

The special materials r coating such as aluminum bronze and stainless steel, which are cavitation resistant materials, should be used.

Effect of cavitation:

The following are the effects of cavitations:

- i) Metallic surfaces are damaged and cavities are formed on the surfaces.
- ii) Due to sudden collapse of vapour bubble, considerable noise and vibrations are produced.
- iii) The efficiency of a turbine decreases due to cavitation and due to pitting action the surface of the turbine blades become rough and the force exerted by water on the turbine blades decreases.

THOMA'S cavitation factor for reaction turbine:

$$\sigma = \frac{H_b - H_s}{H} = \frac{(H_{atm} - H_v) - H_s}{H}$$

H_b -(H_{atm} - H_v) ---- Barometric pressure head in m of water

H_{atm} ----- - atm pressure head in m of water.

H_v ----- Vapour pressure head in m of water.

H_s ----- Suction at the outlet of reaction turbine in m of water. or height of turbine runner above the tail water surface.

H ----- Net head on the turbine in m.

Thoma's cavitation factor σ is compared with critical cavitation factor for the type of turbine.

If $\sigma \geq \sigma_c$ cavitation will not occur.

$$\text{For Francis turbine } \sigma_c = 0.625 \left(\frac{Ns}{380.78} \right)^2$$

$$\sigma = 431 \times 10^{-8} Ns^2$$

$$\text{For propeller turbine } \sigma_c = 0.281 + \left[\frac{1}{7.5} \left(\frac{Ns}{380.78} \right)^3 \right]$$

9) What are pumps and give the formulas used for centrifugal pump?

Hydraulic machines which convert the mechanical energy into hydraulic energy are called pumps.

The hydraulic energy in the form of pressure energy. If the mechanical energy is converted into pressure energy by means of centrifugal force acting on a fluid, the hydraulic machine is called centrifugal pump.

Centrifugal pump acts as a reversed of an inward radial flow reaction turbine. This means that the flow in centrifugal pump is in the radial outward directions.

Main parts of centrifugal pump:

1. Impeller
2. Casing
3. Suction pipe with a float valve and a strainer
4. Delivery pipe.

Centrifugal pump is a reverse of a radially inward flow reaction on turbine:

1. Work done by the impeller on water per
 Second per unit weight of water = Work done increase of turbine
 Striking per second.

$$= \left[\frac{1}{g} (V W_1 U_1 - V W_2 U_2) \right] = \frac{1}{g} [V_{w2} U_2 - V_{w1} U_1]$$

$$= 1/g \ V_{w2} u_2$$

$$2. \text{ Work done per sec} = \frac{w}{g} V_{w2} U_2$$

$$3. W = \text{Weight of the water} = \rho g Q$$

Q = volume of water

$$4. Q = A \times VQ = \pi D_1 B_1 \times V_{f1}$$

$$= \pi D_2 B_2 V_{f2}$$

10) What are the different heads and efficiencies of a centrifugal pump?

Suction head (h_s):

It is the vertical height of the centre line of the centrifugal pump above his water surface in the tank or pump from which water is to be lifted. The height is also called as suction lift and denoted by ' h_s '.

Delivery head (h_d):

The vertical distance between the centre line of the pump and the water surface in the tank to where water is delivered is known as delivery head.

Static head (h_s):

The sum of suction head and delivery head is known as static head.

$$H_s = h_s + h_d$$

Manometric head (h_m):

The manometric head is defined as the head against which a centrifugal pump has to work.

- (i) H_m = Head imported by the impeller to water – loss of head on the pump.
- (ii) $H_m = V_{w2} u_2 / g$ – loss of head in impeller and casing.
- (iii) $H_m = V_{w2} u_2 / g$ -- if loss of pump is zero.

H_m = Total head at outlet of pump –total head at inlet of the pump.

$$H_m = \left(\frac{p_0}{\rho g} + \frac{V_0^2}{2g} + Z_0 \right) - \left(\frac{p_i}{\rho g} + \frac{v_i^2}{2g} + z_i \right)$$

$\frac{p_0}{\rho g}$ ----- Pr.head at outlet of pump = h_d .

$V_0^2/2g$ ----- Velocity head at outlet of pump.

----- Velocity head in delivery pipe= $Vd^2/2g$

Z_0 ----- Vertical head at the outlet of the pump from datum line.

$\frac{p_i}{\rho g}, v_i^2/2g, Z_i$ --- Corresponding values of pressure head, velocity head and datum head at the inlet of pump.

$h_s, V_s^2/2g$ & Z_s respectively.

(c) $H_m = h_s + h_d + h_{fs} + h_{fd} + Vd^2/2g$

Efficiencies of a centrifugal pump:

1. Manometric efficiency
2. Overall efficiency
3. Mechanical efficiency

Manometric efficiency:

$$\eta_{man} = \frac{H_m g}{V w_2 u_2}$$

Overall efficiency:

$$\eta_o = \frac{\frac{(w h_m)}{1000}}{sp}$$

Mechanical efficiency:

$$\eta_m = \frac{\frac{w}{g}}{sp} \left(\frac{V w_2 u_2}{1000} \right)$$